



# Stage 1

## Medium Term Plans

May 2019 (with Unit 15 added)

## Overview of Year

<b>Autumn Term</b>	<b>Number and Algebra</b>			<b>Geometry and Measures</b>	
	1. Investigating Number Systems 2. Pattern Sniffing 3. Solving Calculation Problems	4. Generalising Arithmetic	5. Exploring Shape	6. Reasoning with Measures	

<b>Spring Term</b>	<b>Number and Algebra</b>		<b>Statistics</b>
	7. Discovering Equivalence 8. Reasoning with Fractions	9. Solving Number Problems	10. Investigating Statistics

<b>Summer Term</b>	<b>Geometry</b>	<b>Number and Algebra</b>		<b>Geometry and Measures</b>	
	11. Visualising Shape	12. Exploring Change	13. Proportional Reasoning	14. Describing Position	15. Measuring and Estimating

Stage 1 Year Overview		
Unit	Approx Learning Hours	Summary of Key Content
Introductory Number Unit 1. Investigating Number Systems 2. Pattern Sniffing 3. Solving Calculation Problems	Numbers 1-10: 12 Numbers 1-20: 12 Numbers 1-100: 10  Total of 34	Reading and writing numbers in words and numerals Representing numbers with a range of objects and pictures Counting (forwards and backwards) Counting in 2s, 5s and 10s Number bonds Add and subtract numbers to 20 Read and write mathematical statements
4. Exploring Shape	8	Recognise and name common 2D and 3D shapes
5. Generalising Arithmetic	8	Solve addition and subtraction problems using objects and pictures Read and write mathematical statements to represent these problems
6. Reasoning with Measures	7	Recognise and know the value of money
7. Discovering Equivalence 8. Reasoning with Fractions	8	Recognise, find and name a half and a quarter of an object, shape or quantity Represent half and quarter using different objects and pictures.
9. Solving Number Problems	6	Solve simple multiplication problems, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
10. Investigating Statistics	4	Make and begin to record measurements
11. Visualising Shape	8	Recognise and name common 2D and 3D shapes Begin to describe the properties
12. Exploring Change	8	Sequence events Use the language of dates Tell/show the time to the hour and half hour
13. Proportional Reasoning	8	Solve simple division problems, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher
14. Describing Position	7	Describe position, direction and movement Describe whole, half, quarter and three-quarter turns
15. Measuring and Estimating	6	Compare, describe, measure, begin to record and solve practical problems involving lengths, masses, capacity and time.

Total of 112 hours ~ 23 weeks with 5 hours per week or 28 weeks with 4 hours per week

Stage 1	Introductory Number Unit	
<b>34 learning hours</b>	Units 1-3 : Investigating Number Systems, Pattern Sniffing and Solving Calculation Problems	
	<p>This unit represents a transition from the Reception and EYFS Curriculum to the Year 1-6 National Curriculum. As such the content from 3 standard units has been combined and reordered to support children in moving to larger number systems and ways of working as they develop other learning skills.</p> <ul style="list-style-type: none"> <li>• The Investigating Number Systems unit introduces the number systems and structures that we use at different levels of the curriculum. At KS1 children are working on the place value system of base 10 with the introduction of Roman Numerals as an example of an alternative system in KS2. Negative numbers and non-integers also come in at this stage and progress into KS3. At KS3 and KS4 we start to look at other ways of representing numbers, including standard form, inequality notation and so on.</li> <li>• The Pattern Sniffing unit explores pattern from the early stages of counting and then counting in 2s, 5s, and 10s up to the more formal study of sequences. This sequence work progresses through linear sequences up to quadratic, other polynomial and geometric for the most able older students. Also in this unit children and students begin to study the properties of numbers and to hone their conjecture and justification skills as they explore odd/even numbers, factors, multiples and primes before moving onto indices and their laws.</li> <li>• The Solving Calculation Problems unit explores the concepts of addition and subtraction at KS1, building to wider arithmetic skills including multiplication at KS2. It is strongly recommended that teachers plan this unit for KS1/KS2 with direct reference to the calculation policy. At KS3 students are developing calculation into its more general sense to explore order of operations, exact calculation with surds and standard form (which have been introduced in Inv Number Systems briefly) as well developing their skills in generalising calculation to algebraic formulae.</li> </ul>	
Prior Learning	Core Learning	Learning Leads to...
<ul style="list-style-type: none"> <li>&gt; Children can count reliably with numbers from one to 20, placing them in order and say which number is one more or one less than a given number. (ELG)</li> </ul>	<ul style="list-style-type: none"> <li>&gt; <b>read and write numbers from 1 to 20 in numerals and words.</b></li> <li>&gt; <b>identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least</b></li> <li>&gt; <b>given a number, identify one more and one less</b></li> <li>&gt; <b>count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number</b></li> <li>&gt; <b>count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens</b></li> <li>&gt; <b>represent and use number bonds and related subtraction facts within 20</b></li> <li>&gt; <b>add and subtract one-digit and two-digit numbers to 20, including zero</b></li> </ul>	<ul style="list-style-type: none"> <li>&gt; read and write numbers to at least 100 in numerals and in words</li> <li>&gt; recognise the place value of each digit in a two-digit number (tens, ones)</li> <li>&gt; identify, represent and estimate numbers using different representations, including the number line</li> <li>&gt; compare and order numbers from 0 up to 100; use &lt;, &gt; and = signs</li> <li>&gt; count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward</li> </ul>

- > **read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs**

- > recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
- > recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- > add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- > show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

#### Exemplification

1. Match up these words and numerals: two ten twenty twelve 20 10 2 12
2. a) Using counters, show me: i) 8 ii) 13  
b) Using tens and ones apparatus, show me: i) 14 ii) 6 iii) 20  
c) Show me where these numbers are on a number line: i) 12 ii) 19  
Which one is the most?  
d) Represent 47 using Numicon  
e) Represent 47 using a number line

3. What is one more than 15? What is one less than 58?

4. Say the next three numbers each time:

- a) 24, 25, 26, ....
- b) 36, 37, 38, ....
- c) 78, 77, 76, ....
- d) 103, 102, 101, ...

5. a) Count the stars:

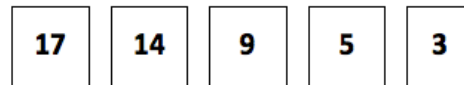


b) Count out 31 counters

c) Read this number out: 36  
Write this number down: 73

c) Carry on this counting:  
5, 10, 15, 20, ....

7. Here are some number cards:



Use the cards to complete four different number facts:

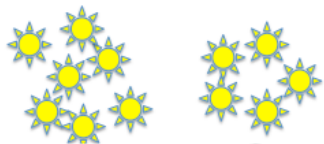
$$\square + \square = \square$$

$$\square + \square = \square$$

$$\square - \square = \square$$

$$\square - \square = \square$$

8. Complete these number sentences:



$$\square + \square = \square$$



$$15 - 6 = \square$$

9. Complete these calculations:

$13 + 5 = \dots\dots$

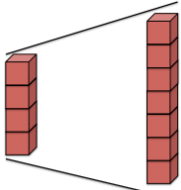
$14 + 0 = \dots\dots$


$7 + \dots\dots = 11$

$16 - 13 = \dots\dots$

Phase 1	Representation	Fluency	Probing Questions
<b>Numbers 1-10</b>  one two three four five six seven eight nine ten zero count next before	<b>Counting</b> <ul style="list-style-type: none"> <li>Using a counting stick to count forwards or backwards from 0-10 or vice versa. Remove some numbers/labels. Count up and down from any point on the stick.</li> <li>Counting aloud using marked number lines, number tracks, 100 squares (top row) to prompt. (Give children some apparatus to use alongside that which you are using with the group e.g. their own number track or their hundred square)</li> <li>Counting objects by moving them initially, then pointing at them with the 'counting finger'</li> </ul>	1. Count up (and beyond) to 10 <ul style="list-style-type: none"> <li>aloud, forwards 0 -10</li> <li>aloud, backwards 10 - 0</li> <li>count a set of up to 10 objects</li> <li>count out a desired number of objects (up to 10)</li> <li>estimate the number of objects and check the answer</li> <li>ext: say the number after/next number for a number up to 10</li> <li>ext: say the number before a number up to 10</li> </ul>	Convince me that 6 comes before 7 when counting forwards  True or False? You can tell how many things there are just by looking?

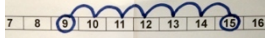
<p>how many? altogether?</p> <p>show tens frame Numicon counters cubes bead string number track unifix number line 100 square</p> <p>numerals words</p> <p>same more less greater than less than &gt; &lt; equal = order smallest largest</p> <p>one more two more three more add plus makes equals</p> <p>one less</p>	<p><b>Recognising Numbers</b></p> <ul style="list-style-type: none"> <li>Developing quick recognition of 1-10 by using tens frame representations</li> <li>Making numbers out of individual objects by counting e.g. counting bears, counters, cubes, animals, children etc.</li> <li>Making numbers out of individual objects that are/can ultimately be grouped into tens and ones e.g. beadstrings, bundles of straws, sticks of unifix cubes</li> <li>Finding and showing numbers on a (marked) number track and then number line</li> </ul>	<p>2. Recognise and represent a number up to 10 using</p> <ul style="list-style-type: none"> <li>Single objects e.g. counters or counting bears or cubes</li> <li>Numicon</li> <li>A bead string</li> <li>A tens frame</li> <li>A number track for 1-10</li> <li>A marked 1-10 number line</li> <li>Base 10 or equivalent (i.e. single cubes for 1-9 and a rod for 10)</li> <li>Ext: Cuisenaire Rods</li> </ul>	<p>Show me how we can represent the number six using</p> <ul style="list-style-type: none"> <li>counting bears</li> <li>unifix cubes</li> <li>the beadstring</li> <li>the cuisenaire rods</li> <li>the numicon</li> <li>the hundred square</li> <li>the number line</li> </ul> <p>Show me where 8 would be on this paper strip that goes from 0-10.</p>
	<p><b>Numerals</b></p> <ul style="list-style-type: none"> <li>Use (and make) a number track to explore the numerals for 1-20</li> <li>Use matching cards between numerals and representations of numbers e.g. Numicon or counters</li> <li>Where number formation is an issue, use tracing/stencil activities (with pens, in sand, on a tablet) to practice correct formation. Also explore the rhyme/<a href="#">song here</a></li> </ul>	<p>3. Read and write numbers up to 10 in numerals</p> <ul style="list-style-type: none"> <li>Read the numbers in numerals aloud e.g. 7, 4</li> <li>Find a given number in numeral form e.g. on a number track or on the wall</li> <li>Match numbers shown using apparatus to numerals e.g. beadstring showing 6 and 6</li> <li>Write a given number in numerals e.g. 9</li> </ul>	<p>What's the same and what's different? 3, 7, 2, 8</p> <p>Show me where you would find the number 9 in real life</p>
	<p><b>Numbers in Words</b></p> <ul style="list-style-type: none"> <li>Use (and make) a washing line to link numbers in words to numbers in numerals to images/representations e.g. Numicon</li> <li>Wherever numbers appear in</li> </ul>	<p>4. Read and write numbers up to 10 in words</p> <ul style="list-style-type: none"> <li>Read the numbers shown in words aloud e.g. two, nine</li> <li>Find a given number in numeral form e.g. on a number track or on the wall</li> <li>Match numbers shown using apparatus to numbers in words e.g. Numicon 3 and 'three'</li> <li>Match numerals and words e.g. 7 and seven</li> </ul>	<p>Show me the number eight in symbols</p> <p>Show me the number 9 in words</p> <p>What's the same and what's different?</p>

two less three less subtract take away (difference)	the classroom, represent them in all three ways if possible e.g. on the clock <ul style="list-style-type: none"> <li>Use matching cards between words, numerals and representations of numbers.</li> <li>When using cards with numbers in words on them, you can place a 'hint' on the back by representing the number using a picture</li> </ul>	<ul style="list-style-type: none"> <li>Begin to write a given number in words e.g. four</li> </ul>	Four; 5; 4; ///, Five; /////
	<b>Comparing and Ordering</b> <ul style="list-style-type: none"> <li>Use a pan balance to help develop the concept of equal – this is particularly effective with Numicon, which is weighted so that it balances when of equal value.</li> <li>Building and then comparing two numbers shown using sticks of Unifix/Multilink to see which is greater (larger) and which is less (smaller).</li> <li>Using sticks e.g. lolly sticks or chop sticks to link the tops and bottoms of these representations to form the &gt; or &lt; signs directly e.g. comparing 4 and 7</li> </ul> 	5. Compare and order numbers <ul style="list-style-type: none"> <li>Compare two numbers to say which is greater</li> <li>Compare two numbers and write &lt; or &gt; to show which is greater</li> <li>Say when two numbers are equal or =</li> <li>Compare three numbers to find the greatest and the least</li> <li>Order three numbers</li> <li>Order four or more numbers</li> <li>Say if an ordering is correct and, if it is not, correct it</li> </ul>	Show me which is greater 7 or 9?  Show me which is fewer (less), 6 or 3?  Show me which of the numbers 8, 5, 7 is the most. (and the least?)  Convince me that 8 is more than 4  Convince me that these numbers are in order of size 4, 7, 8
	<b>More Than</b> <ul style="list-style-type: none"> <li>Sing songs about one more such as, the Turtles song etc. <a href="#">Find songs here</a></li> </ul>	6. Find and show one, two and three more than a number up to 10 <ul style="list-style-type: none"> <li>find one more than a number (0-9)</li> <li>find two more than a number (0-8)</li> <li>find three more than a number (0-7)</li> </ul>	Convince me that 7 is two more than 5  True or False?

<ul style="list-style-type: none"> <li>Building a number practically (e.g. using cubes, a bead string), then adding one/two or three practically before counting again. Some children will start to be able to predict the answer mentally.</li> <li>Finding the number, then jumping on one more (or two more or three more) on a number track or number line or even a 100-square.</li> <li>Using Numicon to make the start number and then attaching a Numicon 1 or Numicon 2 or Numicon 3 to the number before finding the total either visually or by overlaying the piece that fits.</li> </ul>	<ul style="list-style-type: none"> <li>find one/two/three more where answer lies beyond 10</li> <li>ext: find the number you started with if one more than it is ....</li> </ul>	<p>8 is three more than 4</p>
<p><b>Adding</b></p> <ul style="list-style-type: none"> <li>Counting out objects into groups (or hoops etc), combining them and then <u>counting all</u> (good with animals, counting bears, counters, blocks etc)</li> </ul>  <ul style="list-style-type: none"> <li>Counting out objects, then <u>counting on</u> from first number using objects as prompts (usually best to move these objects across one at a time) This can develop to a visual (rather than concrete) approach of adding marks to an existing image to represent the 'extra' ones added as they are counted on.</li> </ul>	<p>7. Add two single digits together (answer less than or equal to 10)</p> <ul style="list-style-type: none"> <li>Find the result of an addition <ul style="list-style-type: none"> <li>one-digit number + 1</li> <li>one-digit number + 2</li> <li>one-digit number + 3</li> <li>one-digit number + 4, 5, 6 etc.</li> <li>one-digit number + 0</li> </ul> </li> </ul> <p>8. Read and complete a number sentence for addition</p> <ul style="list-style-type: none"> <li>Make a number sentence practically e.g. <math>4 + 5 = \dots</math> to solve the problem</li> <li>Complete a number sentence with the answer e.g. <math>4 + 5 = 9</math></li> <li>Ext: Find a missing number in an addition, not crossing 10 e.g. <math>3 + \blacksquare = 8</math> or <math>\blacksquare + 2 = 9</math></li> <li>Put signs into an addition calculation to make it correct</li> </ul>	<p>What's the same and what's different? eight, 8, // // // // //, <math>6 + 2</math>, one more than seven</p> <p>Show me two numbers with a sum of 10</p> <p>Show me two numbers with a sum of 7</p> <p>Convince me that <math>6 + 2</math> gives me the same answer as <math>2 + 6</math></p> <p>Always, Sometimes, Never? If you add zero to a number, the number stays the same.</p>



- Using a number track to find the first number and counting on (or jumping!) the second number (or a hundred square)



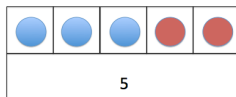
- Using a bead string to count out the first group and then count on the next number to find the total.
- Using Numicon to combine two numbers and overlay with the resulting total value piece.  
e.g.  $7 + 3$



- Using tens frames to represent the first number and then count on the second number.



- A further extension of the tens frame concept can be the use of a 'bus' image to represent the two numbers as people on the bus and new people joining the bus. The bus can have up to 10 seats.
- Counting on mentally, using fingers to represent the second number (e.g. for  $9 + 3$ , "put 9 in your head, then count on 10 [one finger], 11 [two fingers], 12 [three fingers])
- Using a bar model (probably sectioned into blocks) to represent an addition problem.

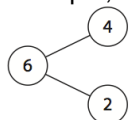


### Number Bonds

- Exploring the pattern of bonds with the same answer e.g. using two different colour counters to show the different ways that you can make 10 by adding two numbers  
This is nice when the counters are arranged in a tens frame. Double-sided counters can work well for this task
- Using a number line or counting stick to find pairs of numbers with the same total – a string number line is excellent for this as you can hold it up to show pairs of numbers with the same total hanging together
- Using a bar model to represent bonds, for example,



- Using a part-part-whole model to represent number facts: for example,



### Less Than

- Sing songs about one less such as 5 Little Speckled frogs, 10 green bottles  
[Find songs here](#)

### 9. Find pairs of numbers that bond to a certain value

- Find bonds to 10
- Find bonds to 5
- Find bonds to 2, 3, 4
- Find bonds to 6, 7, 8, 9
- Given a number, say what bonds with it to make, for example, 9.

Show me all the pairs of numbers that you can find that add up to 9

Show me all the pairs of numbers that you can find that have a difference of 5


Convince me there are lots of pairs of numbers with a difference of 3

### 10. Find and show one, two and three less than a number up to 10

- find one less than a number (1-10)
- find two less than a number (2-10)
- find three less than a number (3-10)
- find one/two/three less where start number lies beyond 10

Convince me that four is three less than seven

True or False?  
0 is three less than 3

<ul style="list-style-type: none"> <li>Building a number practically, then taking one/two/three away practically before counting again. Some children will start to be able to predict the answer mentally.</li> <li>Practising counting backwards as well as forwards so that children are used to dropping back one number and can use this to find one less rapidly.</li> <li>Finding the number, then jumping back one less (or two less or three less on a number track or number line or even a 100-square.</li> <li>Using Numicon to make the start number and then placing a Numicon 1 or Numicon 2 or Numicon 3 over it, before finding the difference either visually or by overlaying the piece that fits in the gap.</li> </ul>	<ul style="list-style-type: none"> <li>e.g. three less than 11</li> <li>ext: find the number you started with if one less than it is ....</li> </ul>	
<p><b>Subtracting</b></p> <ul style="list-style-type: none"> <li>Counting out the first number of objects and taking away the second number of objects by counting them out (usually best to move them away one by one as they are counted). This can extend to a visual method of crossing off the images one by one.</li> </ul> 	<p>11. Subtract a single digit from a number up to 10</p> <ul style="list-style-type: none"> <li>Find the result of a subtraction by taking away or as a difference <ul style="list-style-type: none"> <li>one-digit number/10 - 1</li> <li>one-digit number/10 - 2</li> <li>one-digit number/10 -3</li> <li>one-digit number/10 - 4, 5, 6 etc.</li> <li>one-digit number/10 - 0</li> </ul> </li> </ul>	<p>Show me two numbers with a difference of 4</p> <p>Show me two numbers with a difference of 7</p>
<ul style="list-style-type: none"> <li>Counting out objects into two groups (piles/hoops) and arranging in a line to find the difference between the two</li> </ul>	<p>12. Read and complete a number sentence for subtraction</p> <ul style="list-style-type: none"> <li>Make a number sentence practically e.g. <math>8 - 5 = \dots</math> to solve the problem</li> </ul>	<p>What's the same and what's different? 9-5, 8-4, 10-6, 7-4</p>

sets

e.g. difference between 9 and 6



- Using a number track to find the first number and count back the second number of jumps



- Using a bead string to count out the first group and then count back (take away) the next, beginning to use place value skills to 'read' the resulting value.
- Using Numicon pieces to represent the two numbers and overlaying them – then 'finding' a suitable piece (or possibly pieces if >10) that fill in the gap or difference.



- Using a tens frame to represent the first number and then count back the second number, leaving the answer as the remainder (which should be increasingly quickly visually recognised for its value in this format)
- A further extension of the tens frame concept can be the use of a 'bus' image to represent the two numbers as people on

- Complete a number sentence with the answer e.g.  $8 - 5 = 3$
- Ext: Find a missing number in subtraction, e.g.  $9 - \blacksquare = 7$  or  $\blacksquare - 4 = 2$
- Put signs into an subtraction calculation to make it correct

Always, Sometimes, Never?  
If you take away zero from a number, the number stays the same

the bus and people getting off the bus at a stop. The bus can have up to 10 seats.

- Counting back mentally, using fingers to represent the counts of the second number (e.g. for  $8 - 5$ , "put 8 in your head, then count back 7 [one finger], 6 [two fingers], 5 [three fingers], 4 [four fingers], 3 [five fingers])
- Using a bar model (probably sectioned into blocks) to represent an subtraction problem.

#### Further Extension

1. Compare amounts.

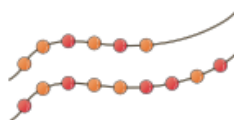
What's the same? What's different?

*Children compare the bead strings and notice:*

*One has 9 beads and the other has 6 beads.*

*9 is 3 more than 6.*

*6 is 3 less than 9.*



2. Fact Families

#### Rich and Sophisticated Tasks

Read and write numbers from 1 to 20 (10) in numerals and words

**NRICH:** [What's in a Name?](#) \*\* I

Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least

**NRICH:** [Making Sticks](#) \*\* P I

**NRICH:** [Dotty Six](#) \* G

Add and subtract one-digit and two-digit numbers to 20 (10), including zero

**NRICH:** [Two Dice](#) \* I

**NRICH:** [Find the Difference](#) \*\* G

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs

**NRICH:** [How Do You See it?](#) \* P

Can you see these number sentences in the picture below?

$$3 + 2 = 5$$

$$2 + 3 = 5$$

$$5 - 3 = 2$$

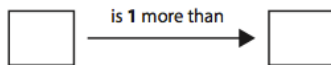
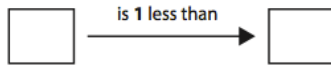
$$5 - 2 = 3$$



Now write the four number sentences for the picture below:



3. Use digit cards to make some true statements with these:



4.

Complete:

$$3 + \square + 3 = 9$$

$$7 + \square + 1 = 10$$

$$6 + 3 + \square = 9$$

$$7 + 1 + \square = 11$$

Represent and use number bonds and related subtraction facts within 20

NRICH: [One Big Triangle](#) \* G

NRICH: [Number Lines](#) \* P

NRICH: [Weighted Numbers](#) \* G P

**Phase 2**

**Numbers 1-20**

eleven  
twelve  
thirteen  
fourteen  
fifteen  
sixteen

**Representation**

**Counting**

- Using a counting stick to count forwards or backwards in 1s from any number from 0-20. Remove some numbers/labels.
- Counting aloud forwards and backwards in 1s using marked number lines, number tracks, 100 squares (top two rows) to prompt. (Give children some

**Fluency**

13. Count up (and beyond) to 20

- aloud, forwards 0 -20
- aloud, backwards 20 - 0
- forwards or backwards from any number lying between 0 and 20
- count a set of up to 20 objects
- count out a desired number of objects (up to 20)
- estimate the number of objects and check the answer
- ext: say the number after/next number for a number up to 20
- ext: say the number before a number up to 20

**Probing Questions**

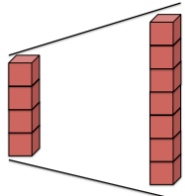
Convince me that 16 comes before 17 when counting forwards


True or False?  
You can tell how many things there are just by looking?

What's the same and what's different?

seventeen eighteen nineteen twenty	apparatus to use alongside that which you are using with the group e.g. their own number track or their hundred square)		Counting and Estimating
tens ones Base 10 Dienes rods	<p><b>Counting in 2s</b></p> <ul style="list-style-type: none"> <li>Using a counting stick to find the multiples of 2 up to 20 and to count forwards and backwards with them</li> <li>Removing numbers from the stick to encourage recall of these key numbers</li> <li>Marking 2s on a number track or 100 square to see the pattern they produce</li> </ul>	<p>14. Count in 2s forwards and backwards up to 20</p> <ul style="list-style-type: none"> <li>Count forwards from 0-20 in 2s</li> <li>Count backwards from 20 to 0 in 2s</li> <li>Say which number is missing in a pattern counting in 2s</li> </ul>	<p>True or False? The number 13 will be said if we count in 2s from 0 to 20</p> <p>Convince me that 16 comes before 18 when counting forwards in 2s.</p>
sum total altogether plus add more	<p><b>Recognising Numbers</b></p> <ul style="list-style-type: none"> <li>Making numbers out of individual objects by counting e.g. counting bears, counters, cubes, animals, children etc.</li> <li>Making numbers out of individual objects that are/can ultimately be grouped into tens and ones e.g. bead strings, bundles of straws, sticks of unifix cubes</li> <li>Making numbers using tens and ones e.g. Numicon, Base 10, Dienes</li> <li>Finding and showing numbers on a (marked) number track/100 square and then number line</li> </ul>	<p>15. Recognise and represent a number up to 20 using</p> <ul style="list-style-type: none"> <li>Single objects e.g. counters or counting bears or cubes</li> <li>Numicon (using a ten + 1-9 piece for numbers over 10)</li> <li>A bead string</li> <li>A number track for 1-20</li> <li>A 100-square</li> <li>A marked 1-20 number line</li> <li>Base 10 or equivalent (i.e. single cubes for 1-9 and a rod for 10)</li> </ul>	<p>Show me how we can represent the number sixteen using</p> <ul style="list-style-type: none"> <li>- counting bears</li> <li>- unifix cubes</li> <li>- the beadstring</li> <li>- the dienes rods</li> <li>- the cuisenaire rods</li> <li>- the numicon</li> <li>- the hundred square</li> <li>- the number line</li> </ul> <p>Show me where 18 would be on this paper strip that goes from 0-20. Where would 8 be?</p>
subtract difference take away minus less fewer number sentence			

<p>track to explore the numerals for 1-20</p> <ul style="list-style-type: none"> <li>Use matching cards between numerals and representations of numbers e.g. Numicon or counters</li> <li>Where number formation is an issue, use tracing/stencil activities (with pens, in sand, on a tablet) to practice correct formation. Also explore the rhyme/<a href="#">song here</a></li> </ul>	<ul style="list-style-type: none"> <li>Find a given number in numeral form e.g. on a number track or on the wall</li> <li>Match numbers shown using apparatus to numerals e.g. beadstring showing 16 and 16</li> <li>Write a given number (read aloud) in numerals e.g. 14</li> </ul>	<p>2, 12, 20</p> <p>Show me where you would find the number 20 in real life</p>
<p><b>Numbers in Words</b></p> <ul style="list-style-type: none"> <li>Use (and make) a washing line to link numbers in words to numbers in numerals to images/representations e.g. Numicon</li> <li>Wherever numbers appear in the classroom, represent them in all three ways if possible e.g. on the clock</li> <li>Use matching cards between words, numerals and representations of numbers.</li> <li>When using cards with numbers in words on them, you can place a 'hint' on the back by representing the number using a picture</li> </ul>	<p>17. Read and write numbers up to 20 in words</p> <ul style="list-style-type: none"> <li>Read the numbers shown in words aloud e.g. eleven, eighteen</li> <li>Find a given number in word form e.g. on the wall, in a book</li> <li>Match numbers shown using apparatus to numbers in words e.g. Numicon 13 and 'thirteen'</li> <li>Match numerals and words e.g. 17 and seventeen</li> <li>Begin to write a given number in words e.g. fourteen</li> </ul>	<p>Show me the number eighteen in symbols</p> <p>Show me the number 15 in words</p> <p>What's the same and what's different? Nine, 19, 0, 9, Nineteen</p>
<p><b>Comparing and Ordering</b></p> <ul style="list-style-type: none"> <li>Use a pan balance to help develop the concept of equal – this is particularly effective with Numicon, which is weighted so that it balances when of equal value.</li> <li>Building and then comparing two numbers shown using</li> </ul>	<p>18. Compare and order numbers up to 20</p> <ul style="list-style-type: none"> <li>Compare two numbers to say which is greater</li> <li>Compare two numbers and write &lt; or &gt; to show which is greater</li> <li>Say when two numbers are equal or =</li> <li>Compare three numbers to find the greatest and the least</li> <li>Order three numbers</li> <li>Order four or more numbers</li> <li>Say if an ordering is correct and, if it is not, correct it</li> </ul>	<p>Show me which is greater 17 or 19?</p> <p>Show me which is fewer (less), 6 or 16?</p> <p>Show me which of the numbers 18, 9, 11 is the most. (and the least?)</p>

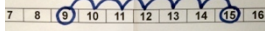
<p>sticks of Unifix/Multilink to see which is greater (larger) and which is less (smaller).</p> <ul style="list-style-type: none"> <li>Using sticks e.g. lolly sticks or chop sticks to link the tops and bottoms of these representations to form the <math>&gt;</math> or <math>&lt;</math> signs directly e.g. comparing 4 and 7</li> </ul> 		<p>Convince me that 11 is more than 4</p> <p>Convince me that these numbers are in order of size 9, 13, 14</p>
<p><b>More Than/Less Than</b></p> <ul style="list-style-type: none"> <li>Sing songs about one more such as 10 Green Bottles, the Turtles song etc. <a href="#">Find songs here</a></li> <li>Building a number practically (e.g. using cubes, a bead string), then adding one/two or three practically before counting again. Some children will start to be able to predict the answer mentally. Removing items for subtraction</li> <li>Finding the number, then jumping on one more (or two more or three more) on a number track or number line or even a 100-square. Jumping back for subtraction</li> <li>Using Numicon to make the start number and then attaching a Numicon 1 or Numicon 2 or Numicon 3 to the number before finding the total</li> </ul>	<p>19. Find and show one, two and three more or less than a number up to 20</p> <ul style="list-style-type: none"> <li>One more</li> <li>Two more</li> <li>Three more</li> <li>One less</li> <li>Two less</li> <li>Three less</li> <li>ext: find the number you started with if one more than it is ....</li> </ul>	<p>Convince me that 17 is two more than 15</p> <p>True or False? 9 is three less than 12</p>

	<p>either visually or by overlaying the piece that fits.</p> <ul style="list-style-type: none"> <li>Using Numicon to make the start number and then placing a Numicon 1 or Numicon 2 or Numicon 3 over it, before finding the difference either visually or by overlaying the piece that fits in the gap.</li> </ul>		
	<p><b>Adding</b></p> <ul style="list-style-type: none"> <li>Counting out objects into groups (or hoops etc), combining them and then <u>counting all</u> (good with animals, counting bears, counters, blocks etc) </li> <li>Counting out objects, then <u>counting on</u> from first number</li> </ul>	<p>20. Add two single digits together (answer greater than 10)</p> <ul style="list-style-type: none"> <li>answer of 11 e.g. <math>8 + 3</math></li> <li>answer of 12 – 15</li> <li>answer of 16-19</li> <li>answer of 20</li> </ul> <ul style="list-style-type: none"> <li>Read, interpret and complete a number sentence e.g. <math>9 + 5 = \dots</math></li> </ul>	<p>What's the same and what's different? <math>6+7, 8+5, 3+10, 4+9</math></p> <p>Convince me that if I count on 5 from 7, I get the same answer as if I counted all of the 5 and the 7</p>

using objects as prompts  
(usually best to move these  
objects across one at a time)  
This can develop to a visual  
(rather than concrete)  
approach of adding marks to  
an existing image to represent  
the 'extra' ones added as they  
are counted on.



- Using a number track to find the first number and counting on (or jumping!) the second number (or a hundred square)



- Using a bead string to count out the first group and then count on the next number to find the total.
- Using Numicon to combine two numbers and overlay with the resulting total value piece.  
e.g.  $7 + 3$



- Using tens frames to represent the first number and then count on the second number.



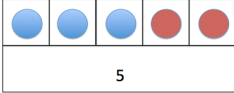
- A further extension of the tens frame concept can be the use of a 'bus' image to represent the two numbers as people on the bus and new people joining the bus. The bus can have up to 10 seats.

21. Add a single digit to a two digit number (answer less than or equal to 20)

- $10 + \text{one digit}$
- $\text{two-digit} + 1$
- $\text{two-digit} + 2$
- $\text{two-digit} + 3$
- $\text{two-digit} + 4, 5, \dots$
- $\text{two-digit} + 0$
- Read, interpret and complete a number sentence e.g.  $12 + 5 = \dots$

Always, Sometimes, Never?  
Addition makes a number larger

Convince me that 18 is four more than 14

	<ul style="list-style-type: none"> <li>Counting on mentally, using fingers to represent the counts of the second number (e.g. for <math>9 + 3</math>, “put 9 in your head, then count on 10 [one finger], 11 [two fingers], 12 [three fingers])</li> <li>Using a bar model (probably sectioned into blocks) to represent an addition problem.</li> </ul> 		
	<p><b>Subtracting</b></p> <ul style="list-style-type: none"> <li>Counting out the first number of objects and taking away the second number of objects by counting them out (usually best to move them away one by one as they are counted). This can extend to a visual method of crossing off the images one by one.</li> </ul>	<p>22. Subtract a single digit from a two digit number</p> <ul style="list-style-type: none"> <li>two-digit – 1</li> <li>two-digit – 2 (not crossing 10)</li> <li>two-digit – 3 (not crossing 10)</li> <li>two-digit – 4, 5, .... (not crossing 10)</li> <li>two-digit – 0</li> <li>10 – single digit</li> <li>two-digit – 2,3, 4, 5, ... (crossing 10)</li> <li>Read, interpret and complete a number sentence e.g. <math>12 - 5 = \dots</math></li> </ul>	<p>Convince me that <math>14 - 5 = 9</math></p> <p>Always, Sometimes, Never? Subtraction makes a number smaller</p>



- Counting out objects into two groups (piles/hoops) and arranging in a line to find the difference between the two sets  
e.g. difference between 9 and 6



- Using a number track to find the first number and count back the second number of jumps



- Using a bead string to count out the first group and then count back (take away) the next, beginning to use place value skills to 'read' the resulting value.
- Using Numicon pieces to represent the two numbers and overlaying them – then 'finding' a suitable piece (or possibly pieces if  $>10$ ) that fill in the gap or difference.




- Using a tens frame to represent the first number and then count back the second number, leaving the answer as the remainder (which should be increasingly quickly visually

23. Subtract a two digit number from a two digit number

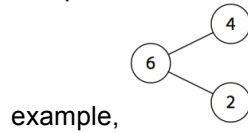
- two-digit  $-10$
- ( $20 - 10$  as a special case of this)
- two-digit  $-$  two-digit
- $20 -$  two-digit
- Read, interpret and complete a number sentence e.g.  $12 - 5 = \dots$

Convince me that  $19 - 16 = 3$

Convince me that the difference between 3 and 7 is the same as the difference between 13 and 17

	<p>recognised for its value in this format)</p> <ul style="list-style-type: none"> <li>• A further extension of the tens frame concept can be the use of a 'bus' image to represent the two numbers as people on the bus and people getting off the bus at a stop. The bus can have up to 10 seats.</li> <li>• Counting back mentally, using fingers to represent the counts of the second number (e.g. for <math>8 - 5</math>, "put 8 in your head, then count back 7 [one finger], 6 [two fingers], 5 [three fingers], 4 [four fingers], 3 [five fingers])</li> <li>• Using a bar model (probably sectioned into blocks) to represent an subtraction problem.</li> </ul>		
	<p><b>Addition and Subtraction Facts</b></p> <ul style="list-style-type: none"> <li>• Exploring the pattern of bonds with the same answer e.g. using two different colour counters to show the different ways that you can make 11 by adding two numbers The above example is nice when the counters are arranged in tens frames. Double-sided counters can work well for this task</li> <li>• Using a bar model to represent bonds, for example,</li> </ul> 	<p>24. Find and use addition and subtraction facts to 20</p> <ul style="list-style-type: none"> <li>• Find numbers that bond to numbers up to 20 <ul style="list-style-type: none"> <li>○ 20</li> <li>○ 11, 12, 13</li> <li>○ 14, 15, 16</li> <li>○ 17, 18, 19</li> </ul> </li> <li>• Give a related subtraction fact for an addition fact e.g. if we know that <math>9 + 5 = 14</math> then <math>14 - 5 = 9</math></li> <li>• Give a related addition fact for a subtraction fact e.g. if we know that <math>19 - 7 = 12</math> then <math>12 + 7 = 19</math></li> <li>• Use a known addition/subtraction fact to find a related fact <ul style="list-style-type: none"> <li>○ two-digit + one digit (using number bonds to 10) [e.g. <math>13 + 6</math> using knowledge of <math>3 + 6</math>]</li> <li>○ two-digit – two-digit (using number bonds to 10) [e.g. <math>17 - 14</math> using knowledge of 3 bonding with 4 to make 7]</li> </ul> </li> </ul>	<p>Always, Sometimes, Never? Addition facts to 20 can be found using addition facts to 10</p> <p>True or False? If I know that <math>13 + 6 = 19</math>, there are 3 other facts that I know.</p>

- Using a part-part-whole model to represent number facts: for



### Further Extension

1. Write the missing number:

19  $\xrightarrow{\text{is 1 less than}}$

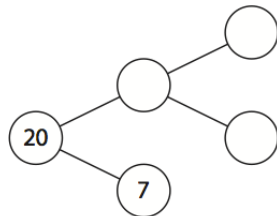
33  $\xrightarrow{\text{is 1 less than}}$

54  $\xrightarrow{\text{is 1 less than}}$

59  $\xrightarrow{\text{is 1 less than}}$

2.

Complete:



Now create a similar diagram.  
Can you extend your diagram?

3.

Write a pair of numbers in the boxes to add to 12.

$$\square + \square = 12$$

And another pair, and another, and another.

Can you find all possibilities? Convince me!

### Rich and Sophisticated Tasks

Read and write numbers from 1 to 20 in numerals and words

**NRICH:** [What's in a Name?](#) \*\* I

Identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least

**NRICH:** [Robot Monsters](#) \* I

**NRICH:** [All Change](#) \* G I

Represent and use number bonds and related subtraction facts within 20

**NRICH:** [Domino Sorting](#) \* I

**NRICH:** [Ladybirds in the Garden](#) \*\* P

**NRICH:** [Pairs of Numbers](#) \* I

**NRICH:** [Butterfly Flowers](#) \* P

Add and subtract one-digit and two-digit numbers to 20, including zero

**NRICH:** [Sort Them Out \(1\)](#) \* G

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs

**NRICH:** [2,4,6,8](#) \*\*\* P

**NRICH:** [What Could It Be?](#) \* I

Phase 3	Representation	Fluency	Probing Questions
<b>Numbers 1-100</b>  thirty forty fifty sixty seventy eighty ninety one hundred  tens ones  2s 5s 10s  first second third fourth fifth sixth seventh eighth ninth tenth	<b>Numbers from 20-100</b> <ul style="list-style-type: none"> <li>Exploring a 100-square to see how the numbers grow and are read. Looking at patterns of tens and ones that occur here</li> </ul>	25. Recognising, saying and beginning to read numbers over 20 <ul style="list-style-type: none"> <li>Counting forwards in 1s to 100 using a prompt e.g. 100 square</li> <li>Counting backwards from a number less than 100 in 1s using a prompt</li> <li>Find the number that comes next using a prompt</li> <li>Find the number that comes before using a prompt</li> </ul>	Always, Sometimes, Never? Numbers with two digits are bigger than numbers with one digit
	<b>Counting in 10s</b> <ul style="list-style-type: none"> <li>Exploring a 100 square to find the tens to help label a counting stick</li> <li>Using a counting stick to show the multiples of 10 and counting these aloud, forwards and backwards</li> <li>Removing some labels from the counting stick to encourage recall of these numbers in sequence</li> <li>Making 10s using Numicon 10s, Base 10 sticks, Dienes rods or bundles of straws.</li> </ul>	26. Recognising and counting in 10s <ul style="list-style-type: none"> <li>count in 10s from zero (forwards)</li> <li>count in 10s from 100 (backwards)</li> <li>say how many an image/apparatus of a multiple of 10 represents</li> <li>find a multiple of 10 on a 100-square</li> <li>count an amount by counting in 10s</li> <li>match the numeral to the number</li> </ul>	Convince me that 40 comes after 30 when counting in 10s.  Show me what comes next: 50, 60, 70, ...  True or false? When you count in 10s, the numbers always end in 0
	<b>Identifying and Representing Numbers</b> <ul style="list-style-type: none"> <li>Making numbers out of individual objects by counting e.g. counting bears, counters, cubes, animals, children etc. e.g. 34 cubes</li> <li>Realising how hard these are to count</li> <li>Then making numbers out of tens and ones equipment e.g., bundles of straws, sticks of unifix cubes, Numicon, base 10 or Dienes blocks</li> <li>Using a 100 bead string to represent numbers using 10s and 1s logic</li> </ul>	27. Show the value of a number up to 100 <ul style="list-style-type: none"> <li>Make a number said aloud using individual objects</li> <li>Make a number written in numerals using individual objects</li> <li>Make a number said aloud using 10s and 1s</li> <li>Make a number written in numerals using 10s and 1s</li> <li>Show a number said aloud on a 100-square or number line</li> <li>Position a number written in numerals on a number line from 0-100</li> </ul>	What's the same and what's different? 45 and 54  What's the same and what's different? 19, 90, 91, 9

<ul style="list-style-type: none"> <li>Finding and showing numbers on a 100 square and then a marked number line</li> </ul>		
<p><b>One More/One Less</b></p> <ul style="list-style-type: none"> <li>Finding a number on a 100-square then jumping on or back one square to find one more/less</li> <li>Making a number using 10s and 1s and then adding/removing 1 to find one more/less</li> <li>Beginning to predict what the number will be using the number system rather than the practical equipment</li> </ul>	<p>28. Finding one more or less than any 2-digit number</p> <ul style="list-style-type: none"> <li>state one more than a 2 digit number (by counting on one) using a prompt</li> <li>state one less than a 2 digit number (by counting back one) using a prompt</li> <li>state one more than a 2-digit number without a prompt</li> <li>state ones less than a 2-digit number without a prompt</li> </ul>	<p>Always, Sometimes, Never? To find 'one more' than a number, just change one digit</p>
<p><b>Comparing and Ordering</b></p> <ul style="list-style-type: none"> <li>Use a pan balance to help develop the concept of equal – this is particularly effective with Numicon, which is weighted so that it balances when of equal value.</li> <li>Building and then comparing two numbers shown using sticks of Unifix/Multilink to see which is greater (larger) and which is less (smaller).</li> <li>Using sticks e.g. lolly sticks or chop sticks to link the tops and bottoms of these representations to form the &gt; or &lt; signs directly e.g. comparing 47 and 39 as two towers</li> </ul>	<p>29. Compare and order numbers up to 100</p> <ul style="list-style-type: none"> <li>Produce representations of two numbers</li> <li>Compare two numbers to say which is greater</li> <li>Compare two numbers and write &lt; or &gt; to show which is greater</li> <li>Say when two numbers are equal or =</li> <li>Compare three numbers to find the greatest and the least</li> <li>Order three numbers</li> <li>Order four or more numbers</li> <li>Say if an ordering is correct and, if it is not, correct it</li> </ul>	<p>Show me which is greater 67 or 59?</p> <p>Show me which is fewer (less), 68 or 38?</p> <p>Show me which of the numbers 54, 39, 23 is the most. (and the least?)</p> <p>Convince me that 54 is more than 51</p> <p>Convince me that these numbers are in order of size 42, 48, 50</p>
<p><b>Counting to and beyond 100</b></p> <ul style="list-style-type: none"> <li>Counting aloud using marked number lines, number tracks,</li> </ul>	<p>30. Counting to and from 100 in 1s without a prompt</p> <ul style="list-style-type: none"> <li>Forwards from 0-100</li> <li>Forwards from any given number</li> </ul>	<p>Convince me that 56 comes after 55 when counting forwards</p>

<p>100 squares and counting sticks to prompt (Give children some apparatus to use alongside that which you are using with the group e.g. their own number track or their hundred square)</p> <ul style="list-style-type: none"> <li>Using a counting stick to start counting at a new value (forwards or backwards)</li> </ul>	<ul style="list-style-type: none"> <li>Backwards from 100</li> <li>Backwards from any given number</li> </ul>	<p>Always, Sometimes, Never? When you count forwards, th</p>
<p><b>Counting in 2s</b></p> <ul style="list-style-type: none"> <li>Using a counting stick to find the multiplies of 2 up to 20 and to count forwards and backwards with them</li> <li>Removing numbers from the stick to encourage recall of these key numbers</li> <li>Marking 2s on a 100 square to see the pattern they produce</li> <li>Represent counting in 5s using repeated addition with:               <ul style="list-style-type: none"> <li>Numicon 2-pieces</li> <li>Counters in groups of 2</li> <li>Placing counter on/Colouring in 100-square</li> </ul> </li> <li>Money (2ps)</li> </ul>	<p>31. Count across 100</p> <ul style="list-style-type: none"> <li>forwards</li> <li>backwards</li> </ul> <p>32. Count in 2s forwards and backwards up to 100</p> <ul style="list-style-type: none"> <li>Count forwards in 2s to 20</li> <li>Count forwards from 0-100 in 2s</li> <li>Count backwards from 100 to 0 in 2s</li> <li>Count on in 2s from any even number</li> <li>Say which number is missing in a pattern counting in 2s</li> </ul>	<p>Show me what comes next: 114, 113, 112, 111, ...</p> <p>True or False? 101 &gt; 99</p> <p>True or False? The number 53 will be said if we count in 2s from 0 to 100</p> <p>Convince me that 46 comes before 48 when counting forwards in 2s.</p>
<p><b>Counting in 5s</b></p> <ul style="list-style-type: none"> <li>Using a counting stick to find the multiplies of 5 up to 50 and to count forwards and backwards with them</li> <li>Removing numbers from the stick to encourage recall of these key numbers</li> <li>Exploring how the values relate e.g. by doubling</li> </ul>	<p>33. Count in 5s forwards and backwards up to 100</p> <ul style="list-style-type: none"> <li>Count forwards in 5s to 50</li> <li>Count forwards from 0-100 in 5s</li> <li>Count backwards from 100 to 0 in 5s</li> <li>Count on in 5s from any multiple of 5</li> <li>Say which number is missing in a pattern counting in 5s</li> </ul>	<p>True or False? The number 56 will be said if we count in 5s from 0 to 100</p> <p>Convince me that 85 comes before 90 when counting forwards in 5s.</p>

<ul style="list-style-type: none"> <li>Marking 5s on a 100 square to see the pattern they produce</li> <li>Represent counting in 5s using repeated addition with:             <ul style="list-style-type: none"> <li>Numicon 5-pieces</li> <li>Counters in groups of 5</li> <li>Bead strings marked in 5s</li> <li>Placing counter on/Colouring in 100-square</li> <li>Money (5ps)</li> </ul> </li> </ul>		
<p><b>Ordinal Numbers</b></p> <ul style="list-style-type: none"> <li>Making and exploring patterns of objects or shapes or numbers and referring the position of the item using an ordinal number</li> <li>Arranging objects according to instructions using ordinals e.g. the first object is a car, the fourth object is a boat etc.</li> </ul>	<p>34. Use ordinal numbers to refer to positions of numbers</p> <ul style="list-style-type: none"> <li>First, second, third</li> <li>Fourth, fifth, sixth</li> <li>Seventh, eight, ninth, tenth</li> <li>11<sup>th</sup> – 20<sup>th</sup></li> <li>20<sup>th</sup> – 100<sup>th</sup></li> </ul>	<p>Convince me that the ninth item comes after the fifth.</p> <p>Always, Sometimes, Never? When you give the position of something, it ends in st, nd or rd e.g. first, second or third</p>
<p><b>Further Extension</b></p>		<p><b>Rich and Sophisticated Tasks</b></p>
<p>1. If Sam places these 5 numbers in order, starting with the smallest number, which number will be in fourth position?</p> <p>46   64   24   42   50</p> <p><input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/></p> <p>smallest <span style="float: right;">largest</span></p>		<p>Count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens</p> <p>NRICH: <a href="#">Writing Digits</a> * P NRICH: <a href="#">Shut the Box</a> * G NRICH: <a href="#">Biscuit Decorations</a> * P NRICH: <a href="#">Grouping Goodies</a> *** P NRICH: <a href="#">Same Length Trains</a> * P</p>

2.  
Alin says, 'If I start at 5 and count in fives I will say the number 100.'  
Is he correct?

Explain your reasoning.

Sita says, 'If I start at 17 and count in twos I will say the number 28.'  
Is she correct?

Explain your reasoning.

3. Look at these digit cards:



Use two of the digit cards to make a number greater than 50.  
Use two of the digit cards to make a number less than 30.  
Use two of the digit cards to make an odd/even number.  
Use two of the digit cards to make a number between 47 and 59.

What is the smallest 2-digit number you can make?  
What is the largest 2-digit number you can make?

### Misconceptions

#### Number Values:

Children sometimes forget about 0 or think that is 'further away' from 1 than 2 is.

Children confuse 0 as a the number with the place holder role - this is true for all numbers e.g. 1 as a ten or as a one

When working beyond 20, beware children confusing tens and teens e.g. fourteen and forty or 14 and 40.

Pupils struggle when counting across 100 to 'begin again' with 101 and also to recall the number prior to 100 when counting backwards.

When counting aloud, children may follow the pattern of the numbers verbally (or

### Teacher Guidance and Notes

#### Numbers 1-10

- When we count, we count in a particular order so you have to say one, two, three, four, five etc rather than one, three, two, five, four, ... This is because we name the size of a set of objects after the last value we counted – the cardinal value of the number. This is a hard concept for children to understand but is critical to their long term number security so allow time for it to develop.
- Teach the model that numbers 0-10 can be represented on their hands but then we need a new thing to count on, hence we pick up a ten marker.

#### Counting

- Moving objects to exemplify secure knowledge of counting is

close to it) but be unable to work with counting in other forms. Listen carefully for incorrect items.

### Place Value

Children find it hard to 'see' numbers above 10 as combinations of 10s and 1s - overcome this by representing them in this way!

Pupils do not always see 'tens' in apparatus based on tens and ones as worth ten of the single units. They may count a 'ten' as just another one.

Children sometimes record numbers backwards e.g. they write 15 as 51 - this can be simply a slip but it can indicate a lack of awareness of the placing of the tens first, then the ones

### Addition and Subtraction

Children struggle to interpret whether to add or subtract from the language used.

When counting on, children may start counting the first extra number using the start number itself, rather than counting the next number (thus they end up with one less than the real answer.

The same is true for subtraction in reverse.

Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships.

Children often do not see difference as a representation of subtraction because take away is emphasised so much. They need to see subtraction represented in this way also to challenge this.

### Number Facts

When working with number facts and bonds, children sometime realise there is a connection e.g.  $3+4=7$  but then incorrectly rearrange this to make a false second fact e.g.  $4 + 7 = 3$

The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'.

beneficial to ensuring place value understanding. You want children to start to 'see' 12 as a '10 and a 2' so use objects to reinforce that representation during counting activities e.g. numicon or base 10 or Dienes.

- Ensure starting numbers for counting on and back are varied.

### Numbers Beyond 10

- When working in place value contexts:
  - encourage children to arrange objects they have counted out into tens and ones
  - represent numbers from 10-20 using a representation that emphasises the place value i.e. show 14 as a 10 and 4 and not just as 14 separate ones. This will help children to begin to understand that when you see a '1' you need to know where it is to interpret its meaning. Beadstrings are a good bridge from a number track to a number line as they maintain the cardinality of the numbers but indicate a continuous approach. They also emphasise the 'specialness' of 10.
  - Don't limit yourself to one preferred representation - encourage children to use all the different equipment to show the numbers they are learning so that they develop a rounder and more secure concept of each number.
  - Vary the resource used here – see the list in the representation box and try to use all of them!
- The article here contains more guidance about developing these concepts for teachers NRICH: [How can I support the development of Early Number Sense and Place Value?](#)

### Addition and Subtraction

- The = symbol should be modelled when exploring numbers that are equal in value.
- This unit is trying to develop the behaviours and understanding of addition and subtraction as well as the notation so make sure you give children opportunities to use practical objects to carry out their calculations as well as tools to record what they have done at the same time.
- In theory, the focus here is on the process of addition and subtraction without context (as this is considered more difficult). In practice, young children often find context easier to deal with than abstract numbers so you will need to judge the level of context to


use. What is deferred until later however, is the application of these ideas to more complex problems so if you are using a context you should keep it simple.

- Try to model addition as both aggregation (finding the total by combining two set) AND augmentation (adding on a number to a set). <https://www.ncetm.org.uk/resources/24134>
- The same is true for subtraction where you should model both finding the difference between two sets and taking away from a set.
- The **calculation policy** has more detailed guidance on how to develop addition and subtraction skills
- The pitch of addition and subtraction work is numbers up to 20, but of course these ideas can be extended beyond 20 for those children who are confident working with numbers between 20 and 100.
- Children need to see and use a variety of question types during this work including: oral questions 'thirteen add four', written questions using symbols ' $11 - 4$ ', simple 'real' problems such as the bus example in representing above or shopping and abstract problems such as finding as many pairs of numbers with a sum of 11 as possible.
- Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction).
- Use 'sum' only to mean an addition calculation – use the word 'calculations' to mean mixed operation computations
- Address confusion caused by the 'how many more' or 'how many fewer' questions that relate to the ordinal values by counting forwards and back on a daily basis, supported with practical resources.
- The recall elements linking to bonds in this unit can be addressed a few at a time - you may want to start teaching children about commutativity so that they don't have to remember all the number facts both ways round!
- Writing addition and subtraction statements using correct notation will be developed further in Unit 5 – for now focus more on reading and understanding.
- Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g.  $? = 4 + 8$  as well as balance problems in Further Extension e.g.  $3 + 4 = ? + 2$

### Key Assessment Checklist

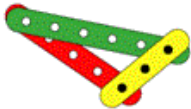

1. I can count up to 10
2. I can recognise and represent a number up to 10 using apparatus
3. I can read and write numbers up to 10 in numerals
4. I can read and write numbers up to 10 in words
5. I can compare and order numbers up to 10
6. I can find and show one, two and three more than a number up to 10
7. I can add two single digits together
8. I can read and complete a number sentence for addition
9. I can find pairs of numbers that bond to a certain value
10. Find and show one, two and three less than a number up to 10
11. I can subtract a single digit from another single digit
12. I can read and complete a number sentence for subtraction
13. I can count up to 20
14. I can count in 2s forwards and backwards up to 20
15. I can recognise and represent a number up to 20
16. I can read and write numbers up to 20 in numerals
17. I can read and write numbers up to 20 in words
18. I can compare and order numbers up to 20
19. I can find and show one, two and three more than a number up to 20
20. I can add two single digits
21. I can add a single digit to a two-digit number (within 20)
22. I can subtract a single digit from a two-digit number (within 20)
23. I can subtract a two-digit numbers from another two-digit number (within 20)
24. I can find and use addition and subtraction facts to 20
25. I can recognise, say and begin to read numbers over 20
26. I can recognise and count in 10s

27. I can show the value of a number up to 100
28. I can find one more or less than any 2-digit number
29. Compare and order numbers up to 100
30. I can count to and from 100 without prompts
31. I can count across 100
32. I can count in 2s
33. I can count in 5s

Stage 1	Unit 4: Exploring Shape																																			
<p><b>8 learning hours</b></p>	<p>In this unit children and students explore the properties of shapes, both 2D and 3D. At KS1 this is focused on common shape names and basic features of vertices, sides etc. but this then develops to classifying quadrilaterals and triangles in KS2. Alongside this focus children begin to explore angle and turn in KS2 and develop this to more formal angle rules through Stages 5, 6, 7, 8. Older students begin to explore the field of trigonometry, encountering first Pythagoras' Theorem, then RA-triangle trig before finally looking at the sine rule and cosine rule.</p>																																			
Prior Learning	Core Learning	Learning Leads to...																																		
<p>DM: 40-60+ months</p> <ul style="list-style-type: none"> <li>➤ Beginning to use mathematical names for 'solid' 3D shapes and 'flat' 2D shapes and mathematical terms to describe shapes</li> <li>➤ Selects a particular named shape</li> </ul> <p>From ELG:</p> <ul style="list-style-type: none"> <li>➤ They [children] explore characteristics of everyday objects and shapes and use mathematical language to describe them.</li> </ul>	<ul style="list-style-type: none"> <li>➤ <b>recognise and name common 2-D and 3-D shapes, including:</b> <ul style="list-style-type: none"> <li>- <b>2-D shapes [for example, rectangles (including squares), circles and triangles]</b></li> <li>- <b>3-D shapes [for example, cuboids (including cubes), pyramids and spheres]</b></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>➤ identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line</li> <li>➤ identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces</li> <li>➤ compare and sort common 2-D and 3-D shapes and everyday objects</li> </ul>																																		
Exemplification		Vocabulary																																		
<p>1. a) Name these shapes</p> <div style="text-align: center;">  </div> <p>b) I am thinking of a 2 dimensional shape. It has four corners. What could my shape be?</p>		<table border="0"> <tr> <td>shape</td> <td>sphere</td> </tr> <tr> <td>2D</td> <td>pyramid</td> </tr> <tr> <td>3D</td> <td>cone</td> </tr> <tr> <td>flat</td> <td>corners vertices</td> </tr> <tr> <td>solid</td> <td>edges sides</td> </tr> <tr> <td>circle</td> <td>faces</td> </tr> <tr> <td>square</td> <td>round</td> </tr> <tr> <td>triangle</td> <td>curved</td> </tr> <tr> <td>rectangle</td> <td>smooth</td> </tr> <tr> <td>pentagon</td> <td>straight</td> </tr> <tr> <td>hexagon</td> <td>roll</td> </tr> <tr> <td>octagon</td> <td>sort</td> </tr> <tr> <td>(oval)</td> <td>find</td> </tr> <tr> <td>(semicircle)</td> <td>draw</td> </tr> <tr> <td>cube</td> <td>name</td> </tr> <tr> <td>cuboid</td> <td>equal same</td> </tr> <tr> <td>cylinder</td> <td></td> </tr> </table>	shape	sphere	2D	pyramid	3D	cone	flat	corners vertices	solid	edges sides	circle	faces	square	round	triangle	curved	rectangle	smooth	pentagon	straight	hexagon	roll	octagon	sort	(oval)	find	(semicircle)	draw	cube	name	cuboid	equal same	cylinder	
shape	sphere																																			
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3D	cone																																			
flat	corners vertices																																			
solid	edges sides																																			
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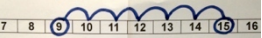
Representation	Fluency	Probing Questions
<p><b>Exploring 2D Shapes</b></p> <ul style="list-style-type: none"> <li>Using shapes to make patterns and describing these</li> <li>Using shapes to make pictures – mosaic pieces, sticky paper shapes, ‘fuzzy felts’. Can they make an animal with a circular head, a triangular body etc... Ask the children to make ‘ruler’ pictures –use a ruler to draw a series of intersecting lines and to colour in each enclosed shape, talking about the properties of these shapes and naming them as they do so.</li> <li>Playing with tangrams to explore the properties of a square. <b>Nrich</b> version</li> <li>Going on a shape hunt to find specific shapes in real life e.g. rectangles</li> <li>Counting (and marking off) the vertices (edges/faces) of a 3D or a 2D shape</li> <li>Drawing round 2D shapes</li> <li>Using the <a href="#">Polygon ITP</a> to explore shapes with ICT</li> </ul>	<p>1. Describe and explore the properties of 2D shapes</p> <ul style="list-style-type: none"> <li>say if the sides are curved or straight</li> <li>say if there are corners</li> <li>say whether it will roll</li> <li>say whether it can be folded in half?</li> <li>begin to count the sides and vertices</li> </ul> <p>2. Use some mathematical language to describe 2D shapes</p> <ul style="list-style-type: none"> <li>name the shape</li> <li>state number of sides</li> <li>state number of vertices</li> <li>begin to say whether any of the sides are equal in/same length</li> </ul>	<p>Show me a shape that will roll</p> <p>Show me a shape with only straight edges</p> <p>Show me a shape that you don't know the name of....</p> <p>Convince me that this shape has 6 vertices</p> <p>What's the same and what's different? triangle and square</p> <p>Always, Sometimes, Never? Triangles have three sides</p>
<p><b>Exploring 3D Shapes</b></p> <ul style="list-style-type: none"> <li>Exploring the feel of 3D (and 2D) shapes to sense whether they are made of rounded faces or flat faces, for example, using a ‘feely bag’</li> <li>Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging!</li> <li>Making models of shapes using plasticine or using construction materials (e.g. blocks, duplo, multi-link etc)</li> <li>Exploring making and ‘unmaking’ 3D shapes using nets/cereal packets etc [can be linked to D&amp;T or other activities e.g. making a box]</li> <li>Printing with 3D shapes to explore the shapes of the faces. Which shapes have</li> </ul>	<p>3. Describe the properties of 3D shapes</p> <ul style="list-style-type: none"> <li>say if the faces are curved (round) or flat</li> <li>say if the edges/sides are curved or straight (or both e.g. cone)</li> <li>say whether it will roll</li> <li>say whether there are corners</li> <li>describe the shape of the faces</li> <li>begin to count the faces and vertices (and possibly edges)</li> </ul> <p>4. Use some mathematical language to describe 3D shapes</p> <ul style="list-style-type: none"> <li>name the shape</li> <li>state the number of faces</li> <li>state the number of vertices</li> <li>state the number of edges</li> <li>state the shapes that make up the faces</li> </ul>	<p>Show me a shape that will roll</p> <p>Show me a shape that can be stacked</p> <p>Show me a shape that you don't know the name of....</p> <p>Convince me that this is a 3D shape</p> <p>Convince me that a cube has 6 faces</p> <p>True or False? A pyramid has more faces than edges</p>

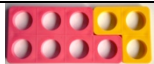
<p>square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder?</p>	<ul style="list-style-type: none"> <li>○ begin to say whether any of or all of the faces and/or edges are the same or equal</li> </ul>	
<p><b>Naming Shapes in different orientations</b></p> <ul style="list-style-type: none"> <li>• Drawing shapes in sand or with chalk outside, then moving to stand and look at these in different positions</li> <li>• Making 2D shapes out of sticks (or children!)</li> <li>• Folding paper in half in different ways. If they make one fold on a square, what shapes can they make? Can they make one fold and make a four-sided shape that isn't a rectangle?</li> </ul>	<p>5. Name 2D and 3D shapes in different orientations</p> <ul style="list-style-type: none"> <li>○ name 2D shapes with base parallel to the bottom of the page/table</li> <li>○ name 3D shapes with base parallel to bottom of page/table</li> <li>○ name 2D shapes when rotated</li> <li>○ name 3D shapes when rotated</li> <li>○ name shapes involved in a combination/compound shape</li> <li>○ name the shape made by putting two known shapes together</li> </ul>	<p>Convince me that this is definitely a cube</p> <p>Always, Sometimes, Never? 2D shapes have 3 sides</p> <p>Convince me that this is a circle</p>
<p><b>Sorting</b></p> <ul style="list-style-type: none"> <li>• Sorting shapes into categories e.g. find all the triangles here</li> <li>• Using hoops to make a Venn diagram to sort shapes out into groups</li> </ul>	<p>6. Sort 2D and 3D shapes into given categories</p> <ul style="list-style-type: none"> <li>○ categories by name e.g. triangles or cubes</li> <li>○ categories by properties e.g. 3D shapes or shapes with 4 vertices or shapes with all straight sides</li> </ul>	<p>Show me how you could sort these shapes into 2 groups - what rule did you use?</p> <p>Show me a shape beginning with c</p> <p>What's the same and what's different? cylinder, cone, circle, sphere</p> <p>What's the same and what's different? square and rectangle</p>
<p><b>Naming shapes from descriptions</b></p> <ul style="list-style-type: none"> <li>• Finding all the shapes that match a given rule e.g. a shape with 6 vertices or a shape with all sides the same length</li> <li>• Playing peekaboo with shapes, trying to guess the shape as more and more is revealed</li> <li>• Playing 'Guess the Shape' from a description</li> </ul>	<p>7. Given a description of a shape, name the shape</p> <ul style="list-style-type: none"> <li>○ situations with only one right answer</li> <li>○ situations where multiple shapes meet the description</li> </ul>	<p>Always, Sometimes, Never? Shapes starting with c can roll</p> <p>Always, Sometimes, Never? A shape with 4 vertices must be 2D</p>

<p>or using a 20 questions format (ensure children get to 'be the teacher' here too)</p> <ul style="list-style-type: none"> <li>Using plastic geostrips to investigate triangles with sides of different lengths</li> </ul> 		
<p><b>Describing shape from the name</b></p> <ul style="list-style-type: none"> <li>Matching the names of shapes to images of the shapes <a href="#">NRich version of shape cards here</a></li> </ul>	<p>8. Given the name of shape, find an example of it and describe its main features</p> <ul style="list-style-type: none"> <li>rectangle, square, triangle, circle</li> <li>pentagon, hexagon, octagon, oval, semicircle etc</li> <li>cube, cuboid, sphere, pyramid</li> <li>cone, cylinder, .... -based pyramid, prism, hemisphere etc.</li> </ul>	<p>Show me what is special about a triangle</p> <p>What's the same and what's different? triangle and circle</p> <p>What's the same and what's different? cube and cuboid</p>
<b>Further Extension</b>		<b>Rich and Sophisticated Tasks</b>
<p>1. What's the same and what's different about these shapes?</p>  <p>Which could be the odd one out and why?</p> <p>Could each one be the odd one out?</p> <p>Explain your reasoning.</p> <p>2.</p> <p>Tom says, 'My shape has 4 rectangular faces and 2 square faces. What is my shape?'</p> <p>Sam says, 'My shape has 2 triangular faces and 3 rectangular faces. How many vertices does my shape have?'</p>		<p>Recognise and name common 2-D and 3-D shapes, including:</p> <ul style="list-style-type: none"> <li>2-D shapes (for example, rectangles (including squares), circles and triangles)</li> <li>3-D shapes (for example, cuboids (including cubes), pyramids and spheres)</li> </ul> <p><b>NRICH: <a href="#">Shaping It</a> * I</b>  <b>NRICH: <a href="#">What's Happening?</a> * P</b>  <b>NRICH: <a href="#">Jig Shapes</a> * P</b>  <b>NRICH: <a href="#">Overlaps</a> ** P</b></p>

Misconceptions	Teacher Guidance and Notes
<p>The most common misconception is a reliance on orientation to identify a shape and hence the failure to recognise a square when it is rotated (as in the exemplification above).</p> <p>Children may confuse flat and solid shapes and fail to see the difference between the two types.</p> <p>Children may want to call curved edges- circle edges. They may also find it hard to work with shapes with both curved and straight edges e.g. semicircle or cone</p> <p>Children may confuse a cube with a square because of the relationship between these- language reinforcement from the start using faces, edges, vertices etc.</p> <p>Squares and rectangles can be confused - bear in mind that later we want children to see a square as a 'special case' of a rectangle</p>	<ul style="list-style-type: none"><li>• The focus of this unit is on developing familiarity with 2D and 3D shapes.</li><li>• Children should be exposed to the mathematical language as much as possible, although they may not mirror this back at first. For example, use language such as equal, vertices, edges etc. wherever possible</li><li>• There is no definitive list of shapes that must be known at this level (although all those in the objective are essential). Therefore, expect questioning to cover common 2D and 3D shapes as listed in the vocabulary box above.</li><li>• There is a school of thought that 3D shapes should be introduced first because they can be handled and are more easily identified in real life (as a 2D shape is a more abstract concept). If you use this approach, ensure you revisit 3D shapes later to use 2D language to describe their faces.</li><li>• It is very important that you vary the orientation of the shapes that you show children so that they do not implicitly 'presume' that the shape has to be a certain way up to qualify as a triangle, for example.</li><li>• Constantly reinforce the properties of these shapes, even though the Stage 1 statements are only about naming and recognising. This is what will define mastery at this stage, rather than simply the ability to name the shapes.</li></ul>
Key Assessment Checklist	
<ol style="list-style-type: none"><li>1. I can recognise and name a circle, rectangle, square and triangle among other shapes</li><li>2. I can recognise and name other 2D shapes among other shapes</li><li>3. I can recognise and name a cube, cuboid, sphere and pyramid among other shapes</li><li>4. I can recognise and name other 3D shapes among other shapes</li><li>5. I can begin to describe the properties of 2D shapes</li><li>6. I can begin to describe the properties of 3D shapes</li><li>7. I can sort shapes into categories I am given</li><li>8. I can explain what is special about a shape</li></ol>	

Stage 1	Unit 5: Generalising Arithmetic																	
<p><b>8 learning hours</b></p>	<p>This unit is focused on developing fluency in the manipulation of number. At primary level this is focused on arithmetic itself and the methods for four operations particularly; however, this is naturally generalised to thinking about rules of arithmetic more widely at secondary level i.e. algebra. These aspects have been paired together intentionally to help teachers describe algebra as simply a generalisation of number. It is expected that teachers will go back to arithmetic to help students see where the 'rules' of algebra come from. Note that the greyed out content is covered elsewhere and hence is not required content here.</p>																	
Prior Learning	Core Learning	Learning Leads to....																
<ul style="list-style-type: none"> <li>Children can count reliably with numbers from one to 20, placing them in order and say which number is one more or one less than a given number. (ELG)</li> <li>Unit 1: Numbers up to 20 (and developed in Unit 2 up to 100)</li> </ul>	<ul style="list-style-type: none"> <li><b>represent and use number bonds and related subtraction facts within 20</b></li> <li><b>add and subtract one-digit and two-digit numbers to 20, including zero</b></li> <li><b>solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = \square - 9</math></b></li> <li><b>read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs</b></li> </ul>	<ul style="list-style-type: none"> <li>recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100</li> <li>add and subtract numbers using concrete objects, pictorial representations, and mentally, including:               <ul style="list-style-type: none"> <li>a two-digit number and ones</li> <li>a two-digit number and tens</li> <li>two two-digit numbers</li> <li>adding three one-digit numbers"</li> </ul> </li> <li>recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> <li>show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot</li> </ul>																
Exemplification		Vocabulary																
<p>1. Find the missing number in each number sentence:</p> <p>a) <math>6 + \blacksquare = 13</math>            b) <math>16 = 5 + \blacksquare</math>            c) <math>17 - \blacksquare = 7</math>            d) <math>12 = 18 - \blacksquare</math></p> <p>2. a) A teacher has 13 children in her classroom. 5 more children come into the room. Write a number sentence to show what happens to the number of children in the classroom.</p>		<table border="0"> <tr> <td>add</td> <td>subtract</td> </tr> <tr> <td>+</td> <td>-</td> </tr> <tr> <td>plus</td> <td>subtraction</td> </tr> <tr> <td>and</td> <td>how many more?</td> </tr> <tr> <td>more</td> <td>take (away)</td> </tr> <tr> <td>make</td> <td>leave</td> </tr> <tr> <td>sum</td> <td>how many left?</td> </tr> <tr> <td>total</td> <td>one less</td> </tr> </table>	add	subtract	+	-	plus	subtraction	and	how many more?	more	take (away)	make	leave	sum	how many left?	total	one less
add	subtract																	
+	-																	
plus	subtraction																	
and	how many more?																	
more	take (away)																	
make	leave																	
sum	how many left?																	
total	one less																	

<p>b) There are 12 eggs in a box. A chef uses 4 eggs to make a cake. Write a number sentence to show what happens to the number of eggs in the box.</p>		<p>altogether score double one more two (ten) more addition equals = is the same as</p>	<p>two less/fewer ten less/fewer how many fewer difference between is the same as minus number sentence</p>
Representation		Fluency	Probing Questions
<p><b>Addition</b></p> <ul style="list-style-type: none"> <li>Counting out objects into groups (or hoops etc), combining them and then <u>counting all</u> (good with animals, counting bears, counters, blocks etc) </li> <li>Counting out objects, then <u>counting on</u> from first number using objects as prompts (usually best to move these objects across one at a time) This can develop to a visual (rather than concrete) approach of adding marks to an existing image to represent the 'extra' ones added as they are counted on. </li> <li>Using a number track to find the first number and counting on (or jumping!) the second number (or a hundred square) </li> <li>Using a bead string to count out the first group and then count on the next number to find the total.</li> <li>Using Numicon to combine two numbers and overlay with the resulting total value piece. e.g. <math>7 + 3</math></li> </ul>		<p>1. Add two numbers to 20 (recap) using equipment as required</p> <ul style="list-style-type: none"> <li>single digit plus single digit – not crossing 10</li> <li>single digit plus single digit – crossing 10</li> <li>two digit plus single digit</li> <li>ext: two digit plus two digit (i.e. crossing 20)</li> <li>adding 0</li> </ul>	<p>Show me two numbers that add up to 20</p> <p>Convince me that 17 is four more than 13</p> <p>What's the same and what's different? <math>5 + 6</math> and <math>8 + 3</math></p> <p>Always, Sometimes, Never? <math>7 + 2</math> is the same as <math>2 + 7</math> (even consider <math>a + b = b + a</math>)</p>



- Counting on mentally, using fingers to represent the counts of the second number (e.g. for  $9 + 3$ , “put 9 in your head, then count on 10 [one finger], 11 [two fingers], 12 [three fingers])

### Subtracting

- Counting out the first number of objects and taking away the second number of objects by counting them out (usually best to move them away one by one as they are counted). This can extend to a visual method of crossing off the images one by one.



- Counting out objects into two groups (piles/hoops) and arranging in a line to find the difference between the two sets e.g. difference between 9 and 6



- Using a number track to find the first number and count back the second number of jumps



- Using a bead string to count out the first group and then count back (take away) the next, beginning to use place value skills to ‘read’ the resulting value.
- Using Numicon pieces to represent the two numbers and overlaying them – then ‘finding’ a suitable piece (or possibly pieces if  $>10$ ) that fill in the gap or difference.




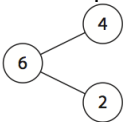
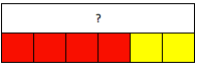
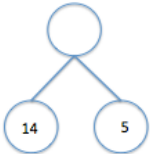
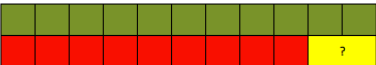
- Counting back mentally, using fingers to

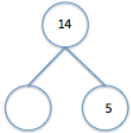
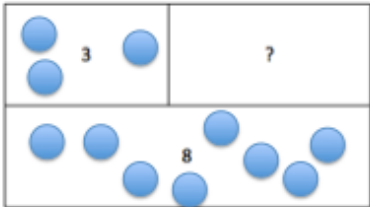
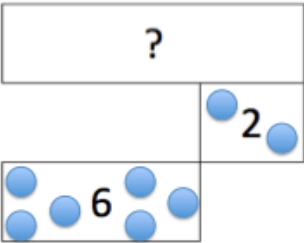
2. Subtract two numbers less than or equal to 20 (recap) using equipment as required

- single digit subtract single digit
- result of 0
- subtracting 0
- two digit subtract single digit – not crossing 10
- two digit subtract single digit – crossing 10
- two-digit subtract two-digit
- special case:  $20 - \blacksquare$

Show me how you can work out  $14 - 5$

Show me two numbers that have a difference of 2

<p>represent the counts of the second number (e.g. for 8 - 5, “put 8 in your head, then count back 7 [one finger], 6 [two fingers], 5 [three fingers], 4 [four fingers], 3 [five fingers])</p>		
<p><b>Related Facts</b></p> <ul style="list-style-type: none"> <li>Using a bar model to represent bonds, for    example, Shows <math>3 + 2 = 5</math> and <math>2 + 3 = 5</math> as well as <math>5 - 3 = 2</math> and <math>5 - 2 = 3</math></li> <li>Using a part-part-whole model to represent    number facts: for example,</li> </ul>	<p>3. Use a known addition/subtraction fact to find a related fact quickly</p> <ul style="list-style-type: none"> <li>two-digit + one digit (using number bonds to 10) [e.g. <math>13 + 6</math> using knowledge of <math>3 + 6</math>]</li> <li>two-digit – two-digit (using number bonds to 10) [e.g. <math>17 - 14</math> using knowledge of 3 bonding with 4 to make 7]</li> </ul>	<p>If I know that <math>14 + 5 = 19</math>, what else do I know?</p> <p>Convince me that <math>19 - 15</math> has the same answer as <math>9 - 5</math></p>
<p><b>Representing Addition Problems</b></p> <ul style="list-style-type: none"> <li>Using a bar model (probably sectioned into blocks) to represent an addition problem. E.g. to represent <math>4 + 2</math>   </li> <li>Alternatively using a part-part-whole model to represent an addition problem   </li> </ul>	<p>4. Interpret a problem as an addition; write the number sentence for this</p> <ul style="list-style-type: none"> <li>problems within 20, complete numbers in a given sentence frame</li> <li>problems within 20, produce whole number sentence</li> <li>ext: problems beyond 20 (may require 10s/1s apparatus)</li> </ul>	<p>Show me how we can find the total of 7 cars and 4 cars</p> <p>Show me a story that you would solve by using this addition <math>16 + 7 = 23</math>, and another...</p>
<p><b>Representing Subtraction Problems</b></p> <ul style="list-style-type: none"> <li>Using a bar model (probably sectioned into blocks) to represent a subtraction problem. E.g. to represent 11 subtract 9   </li> </ul>	<p>5. Interpret a simple problem as a subtraction; write the number sentence for this</p> <ul style="list-style-type: none"> <li>problems within 20, complete numbers in a given sentence frame</li> <li>problems within 20, produce whole number sentence</li> <li>ext: problems beyond 20 (may require 10s/1s)</li> </ul>	<p>Show me how we can find the difference between Joe's 8 teddies and Amy's 15 teddies.</p> <p>Convince me: I have 8 pencils. I take away 3 and there are 5 left.</p> <p>What's the same and what's different?</p>

<ul style="list-style-type: none"> <li>Alternatively, using a part-part-whole model to represent a subtraction problem</li> </ul> 	<p>apparatus)</p>	<p>5 cubes take away 1 cube, and 7 cubes take away 3 cubes</p>
<p><b>Representing missing number problems</b></p> <ul style="list-style-type: none"> <li>Using a bar model and laying objects on it to make the statement true e.g. <math>3 + \blacksquare = 8</math></li> </ul>  <p>e.g. <math>2 \blacksquare - 2 = 5</math></p> 	<p>6. Find missing numbers in simple addition calculations</p> <ul style="list-style-type: none"> <li>addition, not crossing 10, second number missing e.g. <math>3 + \blacksquare = 8</math></li> <li>addition, not crossing 10, first number missing e.g. <math>\blacksquare + 2 = 9</math></li> <li>addition, not crossing 10, answer given first e.g. <math>8 = 2 + \blacksquare</math></li> <li>addition, crossing 10, second number missing e.g. <math>6 + \blacksquare = 13</math></li> <li>addition, crossing 10, first number missing e.g. <math>\blacksquare + 5 = 14</math></li> <li>addition, crossing 10, answer given first e.g. <math>15 = \blacksquare + 7</math></li> </ul> <p>7. Find missing numbers in simple subtraction calculations</p> <ul style="list-style-type: none"> <li>subtraction, below 10, second number missing e.g. <math>8 - \blacksquare = 6</math></li> <li>subtraction, below 10, first number missing e.g. <math>\blacksquare - 2 = 5</math></li> <li>subtraction, below 10, answer given first e.g. <math>5 = 7 - \blacksquare</math></li> <li>subtraction, 2 digits, not crossing 10, second number missing e.g. <math>18 - \blacksquare = 14</math></li> <li>subtraction, 2 digits, not crossing 10, first number missing e.g. <math>\blacksquare - 2 = 13</math></li> <li>subtraction, 2 digits, not crossing 10, answer given first e.g. <math>11 = 17 - \blacksquare</math></li> <li>subtraction, 2 digits, crossing 10, second number missing e.g. <math>13 - \blacksquare = 8</math></li> <li>subtraction, 2 digits, crossing 10, first number missing e.g. <math>\blacksquare - 4 = 7</math></li> <li>subtraction, 2 digits, crossing 10, answer given first e.g. <math>5 = 14 - \blacksquare</math></li> </ul>	<p>What's the same and what's different? <math>3 + \blacksquare = 12</math> and <math>9 + \blacksquare = 12</math></p> <p>Convince me that the missing number is 7</p> $8 + \blacksquare = 15$ <p>Convince me that the missing number cannot be 8:</p> $6 - \blacksquare = 2$ <p>Convince me that the missing number is 13</p> $\blacksquare - 2 = 11$

	<p>8. (Ext) Find missing numbers in combination calculations</p> <ul style="list-style-type: none"><li>• calculations within 10 based on addition e.g. <math>3 + 6 = \blacksquare + 7</math></li><li>• calculations within 20 based on addition e.g. <math>6 + \blacksquare = 9 + 7</math></li><li>• calculations within 10 based on subtraction e.g. <math>9 - 4 = 7 - \blacksquare</math></li><li>• calculations within 20 based on subtraction e.g. <math>\blacksquare - 7 = 11 - 3</math></li><li>• calculations within 10 based on both addition and subtraction e.g. <math>4 + 3 = \blacksquare - 2</math></li><li>• calculations within 20 based on both addition and subtraction e.g. <math>6 + \blacksquare = 19 - 7</math></li></ul>	<p>Show me how you can find this missing number step by step: <math>\blacksquare - 7 = 11 - 3</math></p> <p>True or False? To find a missing number in an add problem, you take away</p>
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**Further Extension**

**Rich and Sophisticated Tasks**

1.  
Robert has 5 more cherries than John.  
John has 11 cherries.  
How many does Robert have?

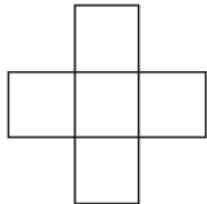
Write a number sentence you would use to solve the problem.

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2.  
Together Sam and Tom have 19 football stickers.  
Tom has 8 stickers. How many stickers does Sam have?

Write a number sentence you could use to solve the problem.

3.  
Write the numbers 1 to 5 in the squares so that each row and column adds up to the same number, called the 'magic number'. What is the 'magic number'?



Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = ? - 9$

**NRICH:** [The Tall Tower](#) \*\*\* P

Read, write and interpret mathematical statements involving addition (+), subtraction (-) and equals (=) signs

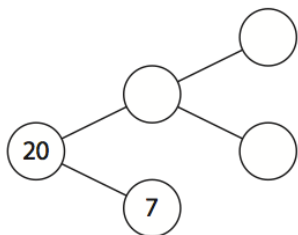
**NRICH:** [2,4,6,8](#) \*\*\* P

**NRICH:** [How Do You See it?](#) \* P

**NRICH:** [What Could It Be?](#) \* I

4.

Complete:



Now create a similar diagram.

Can you extend your diagram?

### Misconceptions

Children struggle to interpret whether to add or subtract from the language used.

When counting on, children may start counting the first extra number using the start number itself, rather than counting the next number (thus they end up with one less than the real answer.

The same is true for subtraction in reverse.

Look out for weak counting in general, which will hold this unit back a great deal.

When working with number facts and bonds, children sometime realise there is a connection e.g.  $3+4=7$  but then incorrectly rearrange this to make a false second fact e.g.  $4 + 7 = 3$

Children can find 'How many more/less?' particularly troublesome as it relates to ordinal values of numbers and relationships.

Children often do not see difference as a representation of subtraction because take away is emphasised so much.

They need to see subtraction represented in this way also to challenge this.

Children forget to include 0 when counting and using number lines – they often see the distance between 0 and 1 as being different to that between other whole numbers (wrongly!).

Children frequently conflate the 'teens' and the 'ties' e.g. mix up fourteen and forty.

### Teacher Guidance and Notes

- This unit is focused on developing the application of addition and subtraction to problems and the recording of this as a number sentence. Earlier work in Unit 3 covers the initial calculation with numbers in a more abstract way and reading mathematical statements.
- However, at this stage it is still important to give children opportunities to use practical objects to carry out their calculations as well as tools to record what they have done at the same time.
- Try to model addition as both aggregation (finding the total by combining two set) AND augmentation (adding on a number to a set). <https://www.ncetm.org.uk/resources/24134>
- The same is true for subtraction where you should model both finding the difference between two sets and taking away from a set.
- The **calculation policy** has more detailed guidance on how to develop addition and subtraction skills
- The pitch of this unit is numbers up to 20, but of course these ideas can be extended beyond 20 for those children who are confident working with numbers between 20 and 100.
- Try to model the wide range of language used to signify addition and subtraction – see vocabulary list above. The children ultimately need to be able to recognise that a problem is an addition problem from the language (and same for subtraction).
- Use 'sum' only to mean an addition calculation – use the word 'calculations' to mean mixed operation computations
- Address confusion caused by the 'how many more' or 'how many fewer' questions that relate to the ordinal values by counting forwards and

The equals sign is not always correctly interpreted as 'has the same value as' by children, who may see it as 'the answer is'.

- back on a daily basis, supported with practical resources.
- Challenge issues with the use of the = sign by looking at examples where the question is on the right e.g.  $? = 4 + 8$  as well as balance problems in Further Extension e.g.  $3 + 4 = ? + 2$

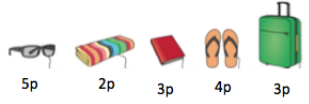

#### Key Assessment Checklist

1. I can use a range of apparatus to add numbers with answers up to 20
2. I can use objects to take away a small number from any number up to 20
3. I can record an addition number sentence and tell you what it means.
4. I can record a subtraction number sentence and tell you what it means.
5. I can solve a problem or puzzle involving addition and subtraction using apparatus or pictures and explain how I did it.
6. I can record my answer to a problem involving addition and subtraction using a number sentence
7. I can find a missing number in an addition problem.
8. I can find a missing number in a subtraction problem.

Stage 1		Unit 6: Reasoning with Measures	
<b>7 learning hours</b>		<p>This unit focuses on mensuration and particularly the concepts of perimeter, area and volume. Primary children are also working on money concepts at this stage, while older secondary students develop mensuration into volume and surface area of challenging shapes, applying Pythagoras' Theorem and trigonometry also in combination with these problems.</p> <p>Note the focus on reasoning within this unit: it is common for children to complete routine problems involving mensuration but this unit is about the developing a secure conceptual understanding of these ideas that they can apply to a wide range of problems and contexts. The opportunity to use and build on earlier number work is built into this unit and it is expected that children apply their arithmetic skills, for example, in these problems.</p>	
Prior Learning	Core Learning	Learning Leads to...	
<ul style="list-style-type: none"> <li>&gt; DM 40-60 months: Beginning to use everyday language related to money</li> <li>&gt; ELG (abridged): Children use everyday language to talk about money to compare quantities and to solve problems.</li> </ul>	<ul style="list-style-type: none"> <li>&gt; <b>recognise and know the value of different denominations of coins and notes</b></li> </ul>	<ul style="list-style-type: none"> <li>&gt; recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value</li> <li>&gt; find different combinations of coins that equal the same amounts of money</li> <li>&gt; solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change</li> </ul>	
Exemplification		Vocabulary	
1. a) Find a 5p coin b) What is the value of this coin? [show a 20p] c) True or false: All coins are silver d) Make £6 in two different ways		money pound penny pence £	p amount coin note
Representation	Fluency	Probing Questions	
<b>Exploring Coins</b> <ul style="list-style-type: none"> <li>• handling coins to learn their values and features e.g. colour, shapes</li> <li>• finding a named coin (show-me task) e.g. find me a 2p</li> <li>• finding a coin to fit a description e.g. find me a coin worth more than 10p</li> <li>• beginning to find coins with a given total e.g. find two coins that make 12p</li> <li>• sorting coins into categories e.g. round and</li> </ul>	1. Recognise and use 1p and 2p coins <ul style="list-style-type: none"> <li>• identify the value given the coin</li> <li>• identify the coin given the value</li> <li>• identify the total value of a set of 1p coins by counting</li> <li>• identify the total value of a set of 2p coins by counting in 2s</li> <li>• identify the value of simple sets of 1p and 2p coins</li> <li>• make a simple amount out of 1p and 2p coins e.g. 7p</li> <li>• ext: find multiple ways to make an amount out of 1p and 2p coins</li> </ul>	What's the same and what's different? 1p, 2p, 3p (2 coins)	



<p>not round or bronze/silver/gold etc.</p> <ul style="list-style-type: none"> <li>playing 'guess my coin' by asking questions such as 'is it round?' or 'is it silver?' or 'is it worth more than 20p?' You can extend this to notes too.</li> </ul>	<p>2. Recognise and use 5p, 10p and 20p coins</p> <ul style="list-style-type: none"> <li>identify the value given the coin</li> <li>identify the coin given the value</li> <li>identify the total value of a set of 10p coins by counting in 10s</li> <li>identify the total value of a set of 5p coins by counting in 5s</li> <li>ext: identify the total value of a set of up to five 20p coins by counting in 20s</li> <li>identify the value of simple sets of 5p, 10p (and 20p) coins e.g. 15p or 25p</li> <li>make a simple amount out of 5p, 10p (and 20p) coins e.g. 35p</li> <li>ext: find multiple ways to make an amount out of 5p, 10p and 20p coins</li> </ul>	<p>What's the same and what's different? 2p, 5p, 20p, 30p (2 coins)</p> <p>Show me a 5p coin</p> <p>Show me a coin that will roll</p>
	<p>3. Recognise and use 50p, £1 and £2 coins</p> <ul style="list-style-type: none"> <li>identify the value given the coin</li> <li>identify the coin given the value</li> <li>identify the total value of a set of £1 coins by counting</li> <li>identify the total value of a set of £2 coins by counting in 2s</li> <li>ext: identify the total value of a set of 50p coins by counting in 50s</li> <li>identify the value of simple sets of £1 and £2 coins e.g. £7</li> <li>identify the value of simple sets of £1 and £2 coins e.g. £4.50</li> <li>make a simple amount out of £1 and £2 coins e.g. £5</li> <li>make a simple amount out of 50p, £1 and £2 coins e.g. £3.50</li> <li>ext: find multiple ways to make an amount out of 50p, £1 and £2 coins</li> </ul>	<p>What's the same and what's different? 2p, 10p, 50p, £1</p> <p>Show me a number of pence that can be made from 2 coins</p> <p>Show me a coin that is worth less than 50p</p>
<p><b>Exploring Notes</b></p> <ul style="list-style-type: none"> <li>handling notes to learn their values and features e.g. colour, sizes</li> <li>finding a named note (show-me task) e.g.</li> </ul>	<p>4. Recognise and use £5, £10, £20 and £50 notes</p> <ul style="list-style-type: none"> <li>identify the value given the note</li> <li>identify the note given the value</li> <li>identify the total value of a set of £10 notes by counting in</li> </ul>	<p>Convince me that a note is worth more than a coin</p>

<p>find me a £5 note</p> <ul style="list-style-type: none"> <li>finding a note to fit a description e.g. find me a coin worth more than £12</li> <li></li> </ul>	<p>10s</p> <ul style="list-style-type: none"> <li>identify the total value of a set of £5 notes by counting in 5s</li> <li>ext: identify the total value of a set of £20 notes by counting in 20s</li> <li>identify the value of simple sets of £10 and £5 notes (and one £20 or £50 note) e.g. £15 or £55</li> <li>make a simple amount out of £10 and £5 notes e.g. £35</li> <li>make a simple amount out of £10, £5, £20 and £50 notes e.g. £25</li> <li>ext: find multiple ways to make an amount</li> </ul>	<p>What's the same and what's different? £5, £10, 5p, 10p</p> <p>What's the same and what's different? 2p, 20p, £2, £20</p> <p>Show me a note worth more than ten pounds</p> <p>Show me a number of pounds for which there is a bank note</p>
<p><b>Using Money</b></p> <ul style="list-style-type: none"> <li>playing shops to practise using coins to pay – and finding the correct amount for simple prices e.g. 2p or £5.</li> <li>Extending to paying for items that cost 1p, 3p, 5p, 7p or 9p using only 2p coins, and receiving the appropriate amount of change in 1p coins.</li> <li>finding the new total when an extra 1p coin is added (linking to one more work earlier) [can also do one less].</li> </ul>	<p>5. Choose the right coin (or note) to pay in a shop (1ps and 2ps)</p> <ul style="list-style-type: none"> <li>give the right number of 1p coins to pay exactly</li> <li>give the right number of 2p coins to pay exactly (prices are even)</li> <li>give the right number of 2p coins to pay and get 1p change (prices are odd)</li> </ul> <p>6. Choose the right coin (or note) to pay in a shop (other coins)</p> <ul style="list-style-type: none"> <li>prices given exactly as a coin amount e.g. 10p or 2p or £1</li> <li>prices given as a multiple of 10p e.g. 30p</li> <li>prices given as a non-exact amount e.g. 4p or 11p requiring 2 coins</li> <li>ext: prices given as a non-exact amount e.g. 4p or 19p requiring giving of one coin (and change)</li> </ul>	<p>Convince me that five 2p coins is worth the same as two 5p coins</p> <p>Show me how to make 7p, 15p and 20p</p> <p>Always, Sometimes, Never? You will get change when you buy something in a shop</p>
<p><b>Equivalence</b></p> <ul style="list-style-type: none"> <li>making a total in different ways with different coins/notes</li> <li>exploring equivalence by counting out a given amount in pennies and then exchanging two pennies for one 2p coin or five pennies for one 5p coin or ten pennies for one 10p coin (build up to the level of your choice)</li> </ul>	<p>7. Find coins that are worth the same</p> <ul style="list-style-type: none"> <li>know that £1 is the same as two 50ps or ten 10ps (ext: or five 20ps)</li> <li>know that a £5 note is the same as five £1 coins (same for other notes)</li> <li>find other coins that are worth the same as 10p, 20p etc.</li> <li>find lots of ways to make a given amount e.g. 11p or £7</li> </ul>	<p>What's the same and what's different? 10p ; 2 x 5ps ; 5p, 2p, 2p, 1p ; 10 x 1ps</p> <p>Always, Sometimes, Never? Money in notes is worth more than money in coins</p>

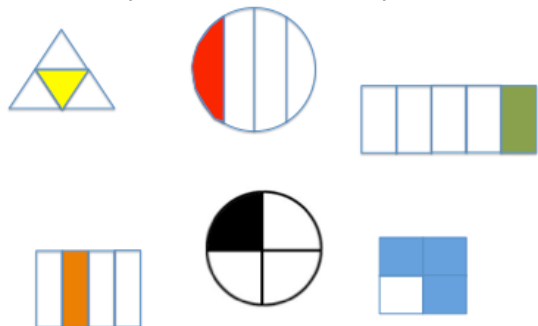
Further Extension	Rich and Sophisticated Tasks
<p>1. How many ways can you find to make 12p?</p> <p>2. Mrs Brown wants to buy a hat for £12. She says “I haven’t got any notes so I can’t buy it”. Is she right or wrong?</p> <p>3. I want to buy these items in a holiday shop:</p>  <p>I have this coin</p>  <p>Have I got enough money?</p>	<p>Recognise and know the value of different denominations of coins and notes</p> <p><b>NRICH:</b> <a href="#">Pirate Shopping (EYFS)</a></p> <p><b>NRICH:</b> <a href="#">Money Bags</a></p>
Misconceptions	Teacher Guidance and Notes
<p>For some children there will be confusion of £ and pence and the distinctions between notes and coins. It may be necessary to reduce the number of coins and notes they are introduced to initially to develop confidence and recall.</p> <p>Some children may have insufficient grasp of number to be able to attribute meaning to each denomination. Counting in 2s, 5s, 10s (and 1s!) is an essential underlying skill to being able to work with denominations of money.</p>	<ul style="list-style-type: none"><li>• In this unit, children are becoming familiar with the coins and notes of the British system. Clearly they bring some external knowledge to this already and should be familiar with numbers up to 20 (from EYFS) and usually higher from the earlier units of this term. The coins are a concrete representation of number - ideally use them in all number units from now on.</li><li>• The pitch of this unit is in knowing the coins and notes and their worth, particularly in terms of their equivalence to other denominations. Work mentioned on change etc is technically an extension of the curriculum but helps make the concepts more meaningful to children.</li><li>• Link counting in 1s, 2s, 5s and 10s into this unit once more by showing images of 1p, 2p, 5p and 10p coins while chanting etc.</li><li>• Use prior knowledge from earlier units of one more/one less and addition and subtraction to support calculations with money, particularly in the shopping context.</li><li>• It is critical to provide lots of experience in exchanging larger denomination coins for single 1p coins. Eg, starting with 23 x 1p coins, a child might group in fives/tens to get 4 x 5p or 2 x 10p plus 3 x 1p or 2p + 1p.</li></ul>



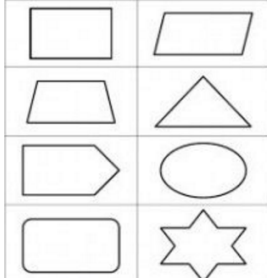
### Key Assessment Checklist



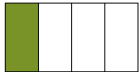

1. I can recognise 1p, 2p coins and know that one 2p coin is equivalent to two 1p coins
2. I can select 1p and 2p coins to make amounts up to 10p
3. I can recognise 5p, 10p coins and know that one 10p coin is equivalent to two 5p coins, five 2p coins or ten 1p coins and that one 5p coin is equivalent to five 1p coins
4. I can recognise 20p, 50p, 100p = £1 coins and know their equivalences in terms of 1p and 10p coins
5. I can select 1p, 2p, 5p and 10p coins to make amounts up to 50p and select 10p, 20 and 50p coins to make multiples of 10p up to £1
6. I know that £1 is equivalent to five 20p or two 50p coins.
7. I can make amounts that are multiples of 10p
8. I can recognise £5, £10 and £20 notes, know their equivalence in terms of £1 coins, and that  $£10 = 2 \times £5$ ,  $£20 = 2 \times £10 = 4 \times £5$

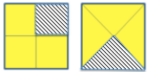
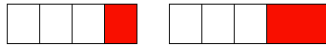

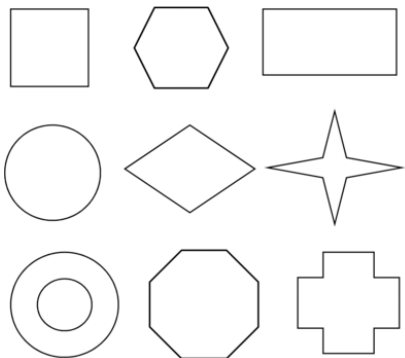

Stage	Unit 7/8: Discovering Equivalence/Reasoning with Fractions	
<p><b>8 learning hours</b></p>	<p>This unit is a combination of two units that are separated in older year groups to allow teachers extra time to master the concepts. The unit explores the concepts of fractions (decimals and percentages) as ways of representing non-whole quantities and proportions. For the youngest children, the work is focused on fractions and developing security in recognising and naming them. At KS2 this then builds to looking at families of fractions and decimals and percentages and fraction arithmetic. At secondary level this is extended to more complex percentage work and equivalence with recurring decimals and surds.</p>	
Prior Learning	Core Learning	Learning Leads to...
<p>&gt; ELG (abridged): Children solve problems including doubling, halving and sharing</p>	<p>&gt; <b>recognise, find and name a half as one of two equal parts of an object, shape or quantity</b></p> <p>&gt; <b>recognise, find and name a quarter as one of four equal parts of an object, shape or quantity</b></p> <p>&gt; <b>(identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least)</b></p>	<p>&gt; recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a length, shape, set of objects or quantity</p> <p>&gt; write simple fractions for example, <math>\frac{1}{2}</math> of 6 = 3 and recognise the equivalence of <math>\frac{2}{4}</math> and <math>\frac{1}{2}</math></p> <p>&gt; identify, represent and estimate numbers using different representations, including the number line</p>
Exemplification		Vocabulary
<p>1. a) Put a circle round on half of these sweets:</p>  <p>b) Represent <math>\frac{1}{2}</math> of this square in 3 different ways:</p> 		<p>half halve equal parts whole all share group dividing quarter fourth share equally equal groups equal to</p>

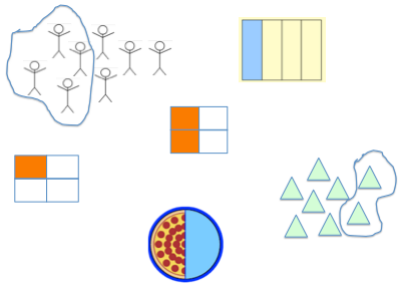
2. Tick the pictures that show a quarter shaded:



Representation	Fluency	Probing Questions
<p><b>Fractions of an object or shape: <math>\frac{1}{2}</math></b></p> <ul style="list-style-type: none"> <li>Saying how many parts are in the whole and how many are shaded</li> <li>Colouring in 1 part out of 2 in given shapes to find <math>\frac{1}{2}</math></li> <li>Specifically folding (and colouring) paper strips to show <math>\frac{1}{2}</math> (precursor to the bar model)</li> </ul>  <ul style="list-style-type: none"> <li>Finding different ways to fold a post-it to show <math>\frac{1}{2}</math></li> <li>Manipulating fraction pieces (bars and/or circles)</li> </ul>	<p>1. Recognise one half:</p> <ul style="list-style-type: none"> <li>state the number of parts in the whole (of a shape/set of objects)</li> <li>know that these parts must be of equal size (and say when they are not)</li> <li>state the number of parts that are shaded</li> <li>say whether a shaded section of a whole shape is one half or not</li> <li>say whether a shaded section of a shape split into equal parts is one half or not</li> <li>write the fraction of a shape that has been shaded i.e. <math>\frac{1}{2}</math></li> <li>say whether a group from a set of objects is one half of the whole (by checking that each group is of the same size)</li> </ul>	<p>Convince me that the red shape is half of the whole shape</p> 
<p><b>Half of a shape</b></p> <ul style="list-style-type: none"> <li>Folding (and colouring) a range of symmetrical paper shapes in half to find half of the shape</li> <li>Splitting a shape into two equal pieces and shading one of the two</li> </ul>	<p>2. Find half of a shape</p> <ul style="list-style-type: none"> <li>find half of a shape with two marked equal parts of same shape</li> <li>find half of a shape by splitting it into two equal parts (in different ways if appropriate)</li> <li>find half of a shape with four marked equal parts</li> <li>find half of a shape with six, eight or ten marked equal parts</li> <li>ext: find half of a shape with marked equal parts of different shapes</li> </ul>	<p>Show me half of these shapes</p> 

	<p>e.g. </p>	
<p><b>Fractions of quantities: <math>\frac{1}{2}</math></b></p> <ul style="list-style-type: none"> <li>sharing out discrete objects (e.g. counters, multilink, animals, counting bears, pencils, frogs) into two containers (e.g. buckets, hoops, lily-pads) and counting the number in each container</li> <li>grouping discrete objects into 2s and counting the number of groups created to find a half</li> <li>using a bead string to count out the number and share it into 2 equal groups to find the value of a half of the number</li> <li>checking whether a quantity has been halved by counting each group to check they are equal</li> </ul>	<p>3. Find <math>\frac{1}{2}</math> of a number of objects</p> <ul style="list-style-type: none"> <li>find half of a set of objects (small even number) by sharing them into two containers and counting the contents of one container</li> <li>find half of a set of objects (small even number) by grouping them into 2s and counting the number of groups</li> <li>find half of a set of drawn objects (even number) by sharing them into two groups and counting the number of objects in each group</li> <li>find half of a set of drawn objects (even number) by grouping them in 2s and counting the number of groups</li> <li>know that if objects are in two groups with the same number in each group, then each group represents <math>\frac{1}{2}</math></li> <li>ext: recognise that when you find half of a set of objects where there are an odd number, you will be left with one object</li> </ul>	<p>Show me half of this group of strawberries</p>  <p>Show me how you can find half of these counters by sharing them into two groups</p> <p>Show me how you can find half of these counters by putting them into groups of two</p> <p>What's the same and what's different? Sharing and grouping</p> <p>Always, Sometimes, Never? Half of zero is zero</p>
<p><b>Fractions of an object or shape: <math>\frac{1}{2}</math></b></p> <ul style="list-style-type: none"> <li>Colouring in 1 part out of 4 in given shapes to find <math>\frac{1}{4}</math></li> <li>Folding (and colouring) paper strips to show <math>\frac{1}{4}</math> (precursor to the bar model)</li> </ul>  <ul style="list-style-type: none"> <li>Finding different ways to fold a post-it to show <math>\frac{1}{4}</math></li> </ul>	<p>4. Recognise one quarter:</p> <ul style="list-style-type: none"> <li>state the number of parts in the whole (of a shape/set of objects)</li> <li>know that these parts must be of equal size (and say when they are not)</li> <li>state the number of parts that are shaded</li> <li>say whether a shaded section of a shape split into four equal parts is one half or not</li> <li>say whether a shaded section of a whole shape is one quarter or not</li> <li>say whether a shaded section of a shape split into more</li> </ul>	<p>Convince me that the green section is not a quarter of the whole shape</p>  <p>What's the same and what's different?</p>

 <ul style="list-style-type: none"> <li>Manipulating fraction pieces (bars and/or circles)</li> </ul>	<p>equal parts (e.g. eight parts) is one quarter or not</p> <ul style="list-style-type: none"> <li>write the fraction of a shape that has been shaded i.e. <math>\frac{1}{4}</math></li> <li>say whether a group from a set of objects is one quarter of the whole (by checking that each group is of the same size)</li> <li>say whether a group from a set of objects is one quarter of the whole (by splitting the remaining objects up into groups of the same size and checking there are four equal parts)</li> </ul>	 <p>What's the same and what's different? Half and quarter</p>
<p><b>Quarter of a shape</b></p> <ul style="list-style-type: none"> <li>Folding (and colouring) a range of symmetrical paper shapes in half and then in half again to find a quarter of the shape</li> <li>Splitting a shape into four equally sized pieces and shading one of the four</li> </ul>	<p>5. Find and recognise a quarter of a shape</p> <ul style="list-style-type: none"> <li>find a quarter of a shape with four marked equal parts of same shape</li> <li>find a quarter of a shape by splitting it into four equal parts (in different ways if appropriate)</li> <li>find a quarter of a shape with eight marked equal parts</li> <li>ext: find a quarter of a shape with twelve, sixteen or twenty marked equal parts</li> <li>ext: find a quarter of a shape with marked equal parts of different shapes</li> </ul> <p>e.g. </p>	<p>Show me one quarter of these shapes</p> 
<p><b>Fractions of quantities: <math>\frac{1}{4}</math></b></p> <ul style="list-style-type: none"> <li>sharing out discrete objects (e.g. counters, multilink, animals, counting bears, pencils, frogs) into four containers (e.g. buckets, hoops, lily-pads) and counting the number in each container</li> <li>grouping discrete objects into 4s and counting the number of groups created to find a quarter</li> <li>using a bead string to count out the number and share it into 4 equal groups to find the value of a quarter of the number</li> <li>checking whether a quantity has been quartered by counting each group to check they are equal</li> </ul>	<p>6. Find and recognise <math>\frac{1}{4}</math> of a number of objects</p> <ul style="list-style-type: none"> <li>find a quarter of a set of objects (small multiple of 4) by sharing them into four containers and counting the contents of one container</li> <li>find a quarter of a set of objects (small multiple of 4) by grouping them into 4s and counting the number of groups</li> <li>find a quarter of a set of drawn objects (small multiple of 4) by sharing them into four groups and counting the number of objects in each group</li> <li>find a quarter of a set of drawn objects (small multiple of 4) by grouping them in 4s and counting the number of groups</li> <li>know that if objects are in four groups with the same number in each group, then each group represents <math>\frac{1}{4}</math></li> </ul>	<p>Show me one quarter of this group of bananas</p>  <p>Show me how you can find a quarter of 16 in two different ways</p>

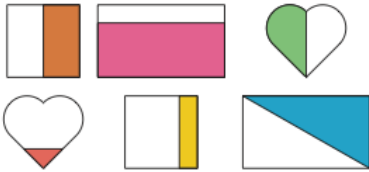
	<ul style="list-style-type: none"> <li>ext: recognise that sometimes there will be objects leftover when trying to find a quarter</li> </ul>	<p>Convince me that a quarter of 12 must be 3</p> <p>Always, Sometimes, Never? Sharing is quicker than grouping</p>
<p><b>Representing Fractions</b></p> <ul style="list-style-type: none"> <li>Using bar models, shapes, Numicon, Cuisenaire rods etc. to represent fractions</li> <li>Matching images of fractions that represent <math>\frac{1}{2}</math> or <math>\frac{1}{4}</math></li> </ul>	<p>7. Represent one half and one quarter with a range of different concrete apparatus and pictorial images</p> <p>8. Solve problems involving <math>\frac{1}{2}</math> and <math>\frac{1}{4}</math></p>	<p>What's the same and what's different?</p>  <p>Always, Sometimes, Never? You cannot find half of 7</p> <p>Convince me that I get the same result when I halve by sharing as I do when I halve by grouping</p> <p>Convince me that a quarter of 16 is the same as half of 8</p> <p>What's the same and what's different? Half of 10 cubes; one quarter of 20 cubes</p> <p>Always, Sometimes, Never? When you fold a square in half and in half again, you get another square.</p> <p>Always, Sometimes, Never? When I find half of a number, I get a bigger answer than if I find a quarter of the same number</p>

**Further Extension**

1.  
Which of these show half of each whole shape?

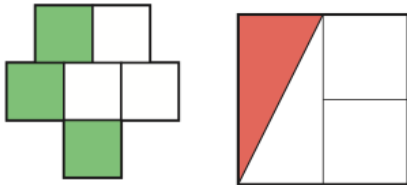
Explain your reasoning.

*Children should talk about the two parts needing to be equal parts of the whole.*

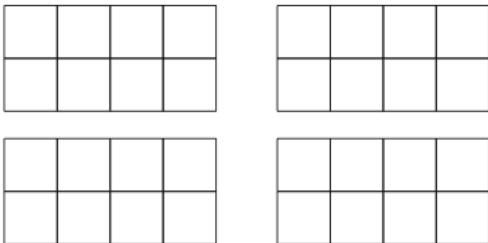


2.  
What fraction of the whole shape is shaded?

Explain your reasoning.



3.  
Shade each whole shape to show half in four different ways.



4.  
What is half of this amount?



**Rich and Sophisticated Tasks**

Recognise, find and name a half as one of two equal parts of an object, shape or quantity

NRICH: [Halving](#) \*\* I  
NRICH: [Happy Halving](#) \*\*\* P  
NRICH: [Fair Feast](#) \* P

**Other Problems**

1.  
Sam and Tom share the fruit equally. There are 4 apples, 3 oranges, 1 pear and 1 banana.  
How many of each fruit do they receive?  
Complete the table below.



	Apples	Oranges	Bananas	Pears
Sam				
Tom				

[nctm.org.uk/resources/42634](http://nctm.org.uk/resources/42634) ppt slides  
The Lighthouse Keeper's Lunch activities  
[www.nctm.org.uk/public/files/6239802/Activity-1.pdf](http://www.nctm.org.uk/public/files/6239802/Activity-1.pdf)

5.  
Half the children at a party are girls.  
How many children could be at the party?  
Give four different answers.

Explain your reasoning.

6.  
Four children share 2 pizzas equally. Draw a diagram to show how much pizza each child gets.  
What fraction of the pizzas does each child eat?

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Four children share two bags of 8 marbles equally. Draw a diagram to show how many marbles each child gets.  
What fraction of one bag of marbles does each child get?

7.  
Complete this halving wall.  
What is the relationship between the top row and one part of your final row?  
Explain your reasoning.

20			
10			

Choose any number and create your own halving wall.

### Misconceptions

Children think a half of a shape has to be represented with a vertical cut i.e. a left and right piece

Children do not realise the importance of dividing a shape/object into EQUAL parts to find a half or a quarter - they may divide it into two or four unequal parts and claim that one of them is a half or a quarter.

Note that it is not essential for each part to be the same shape (only the same size) and some children find this difficult to accept. For example, this diagram does show quarters of the whole although the four parts are not all the same shape. For example, this shape has four equal parts (in terms of size):

### Teacher Guidance and Notes

- This unit represents children's first introduction to the concept of fractions.
- They may have some experience of sharing and halving from previous learning in some cases but this will be without formal reference to fractions.
- It is crucial to ensure that children are using the language of whole and parts when talking about fractions. You will need to model this description of a fraction and explanation.

For example, for this square:



“ if the square is the whole, one part is one quarter of the whole square”



Children tend to focus on the sharing model for halving and quartering (i.e. dealing out into 2 or 4 groups) and do not think about the grouping model as an alternative (which is what they will need later for larger scale division and fractions).

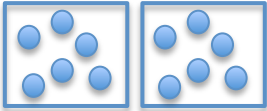
Children stick to one particular representation of a half or a quarter (often the circular one) and do not recognise other models or images as also being a half or a quarter.





Encourage children to use this language themselves when explaining their thinking.

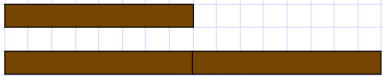

- Children will need to experience finding half then quarter with a variety of practical resources and related to real life experiences before moving to pictorial and symbolic representations. Try to use as many different objects, shapes and quantities as you can when working to find a half - begin to distinguish between finding a half of an object by dividing it into EQUAL parts and finding half of a quantity or number by sharing or grouping.
- Later work with division and fractions needs children to understand the TWO processes of sharing and grouping as different ways for dividing - make sure you match your language appropriately so that children understand the difference between the two approaches and that they should give the same answer!
- Recognising the quantity in a set (subitising) is an important step to efficient calculation and estimation. It is, for example, recognising the pattern of six on a dice without counting. This can be built into this unit to help children quickly count before halving.
- When finding a quarter of an amount, draw attention to the division into four equal parts (and note that children will often only hear 'four parts' here and forget about the need for them to be equally sized) as well as to the relationship between a quarter and a half.
- As you move to finding a quarter of a quantity or number, again make reference to sharing and to grouping as different structures for carrying out this process.


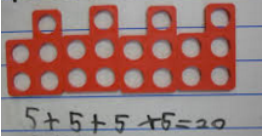
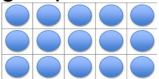
#### Key Assessment Checklist

1. I can say if a shaded part is half of a shape or not.
2. I can show half of a shape or object by dividing it into two equal parts.
3. I can find half of a small number of objects by sharing them into two equal groups and counting the objects in one of the groups.
4. I can find half of a small number of objects by grouping them into 2s and counting how many groups I have.
5. I can represent a half using a range of models and images.
6. I can say if a shaded part is a quarter of a shape or not.
7. I can show a quarter of a shape or object by dividing it into four equal parts.
8. I can find a quarter of a small number of objects by sharing them into four equal groups.
9. I can find a quarter of a small number of objects by grouping them into 4s and counting how many groups I have.
10. I can represent a quarter using a range of models and images, including as half of a half.

Stage 1		Unit 9 : Solving Number Problems	
<b>6 learning hours</b>	<p>This unit continues pupils' earlier study of arithmetic (and algebra for secondary students). At Key Stage 1 children are working on multiplication (and division in Stage 2) as a way to represent repeated addition and scaling (and repeated subtraction – grouping - and sharing). At Key Stage 2 children are developing skills in applying their arithmetic to more complex problems. At secondary level and in Stage 6, students begin to find unknown values by applying inverse operations. Equations of all types including quadratic and simultaneous are covered in later stages.</p>		
Prior Learning	Core Learning	Learning Leads to...	
<ul style="list-style-type: none"> <li>➤ 40-60+ months: Children find the total number of items in two groups by counting all of them.</li> <li>➤ ELG: Children solve problems, including doubling, halving and sharing.</li> </ul>	<ul style="list-style-type: none"> <li>➤ <b>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</b></li> </ul>	<ul style="list-style-type: none"> <li>➤ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (<math>=</math>) signs</li> <li>➤ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot</li> <li>➤ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts</li> </ul>	
Exemplification		Vocabulary	
<p>1. a) What is double 8? b) I have 2 boxes of counters. There are 5 counters in each box. How many counters are there altogether?</p>  <p>c) Bob has 3 bags of sweets. Each bag has 4 sweets in. How many sweets does Bob have altogether? d) I have 6 pairs of socks. How many socks do I have altogether? e) Work out 3 multiplied by 5 f) Calculate <math>2 \times 7</math></p>		<p>multiplication multiplied by multiply times sets of lots of groups of per scaling twice</p>	<p>... times as ... double (treble) pairs group array how many altogether? represent</p>
Representation	Fluency	Probing Questions	
<p><b>Multiplication by 2</b></p> <ul style="list-style-type: none"> <li>• Arranging objects into equal groups and counting efficiently in 2s</li> </ul>	<p>1. Multiply numbers by 2 in context</p> <ul style="list-style-type: none"> <li>• count in 2s from 0</li> <li>• find the total of objects arranged in pairs by counting</li> </ul>	<p>Show me 3 groups of 2  Show me the amount twice as big as</p>	


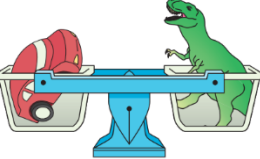
<p>(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on) e.g. 2 frogs on each lily pad or 5 shapes in each hoop</p> 	<p>in 2s (e.g. socks on a washing line, Numicon 2s)</p> <ul style="list-style-type: none"> <li>• arrange objects into groups of 2 to count them</li> <li>• given one set of objects, find the numbers in 2 sets by building a second set and counting the total</li> <li>• given a word problem, represent it with objects and count the total e.g. I have 2 groups of 6 counters. How many counters do I have altogether?</li> </ul>	<p>10</p> <p>Convince me that if I have 7 pairs of socks then I have 14 socks altogether.</p>
<ul style="list-style-type: none"> <li>• Using Numicon 2s (or unifix cubes or other attached materials) to represent equal groups of 2 and counting the total by counting efficiently in 2s</li> <li>• Using a bead string to represent groups of 2 (i.e. repeated addition) and count the total (using place value to 'read' the total more quickly if possible)</li> </ul>  <ul style="list-style-type: none"> <li>• Pegging pairs of socks on to a washing line</li> </ul>  <ul style="list-style-type: none"> <li>• Saying the multiplication that has been made. For example, for <math>2 \times 4</math> we would expect to see groups of 2 shown four times and children to practise reading and saying the multiplication as they are building it '4 lots of 2' and '4 groups of 2'.</li> <li>• Using an array (with help) to structure the groups of 2 more formally e.g. <math>2 \times 3</math></li> </ul>  <ul style="list-style-type: none"> <li>• Using Cuisenaire rods (or straws) to scaling an amount/length by making it twice as big/doubling it/making it ten times bigger and so on. For example, doubling 8</li> </ul>	<p>2. Multiply numbers by 2 more abstractly</p> <ul style="list-style-type: none"> <li>• vocalise a multiplication and know that 2 multiplied by 6, for example, means 6 groups of 2</li> <li>• represent an abstract multiplication by 2 with a range of equipment and count the total e.g. 2 multiplied by 4 or 3 multiplied by 2</li> <li>• produce an array to represent a multiplication</li> <li>• double a number by counting out two identical groups and finding the total e.g. double 7</li> <li>• recognise a range of words implying 'multiply by 2' and solve problems involving these</li> </ul>	<p>Show me how you show 2 multiplied by 3 in as many ways as possible</p> <p>Show me the array for 5 groups of 2</p> <p>Convince me that twice 6 is 12</p> <p>Convince me that 8 lots of 2 is 16</p>

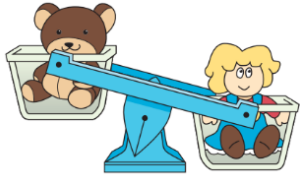
 <ul style="list-style-type: none"> <li>Using 2p coins to represent 2s particularly and encourage counting in these denominations</li> </ul>		
<p><b>Multiplication by 10</b></p> <ul style="list-style-type: none"> <li>Arranging objects into equal groups and counting efficiently in 10s (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)</li> <li>Using Numicon 10s (or unifix cubes or other attached materials) to represent equal groups of 10 and counting the total by counting efficiently in 10s</li> <li>Saying the multiplication that has been made. For example, for <math>10 \times 4</math> we would expect to see groups of 10 shown four times and children to practise reading and saying the multiplication as they are building it '4 lots of 10' and '4 groups of 10'.</li> <li>Using an array (with help) to structure the groups of 0 more formally e.g. <math>10 \times 3</math></li> </ul>  <ul style="list-style-type: none"> <li>Using Cuisenaire rods (or straws) to scaling an amount/length by making it ten times bigger and so on.</li> <li>Using coins to represent 10s particularly with 10ps and encourage counting in these denominations</li> </ul>	<p>3. Multiply numbers by 10</p> <ul style="list-style-type: none"> <li>count in 10s from 0</li> <li>find the total of objects arranged in tens by counting in 10s (e.g. sticks of 10 unifix, Numicon 10s)</li> <li>arrange objects into groups of 10 to count the total</li> <li>given a word problem, represent it with objects and count the total e.g. I have 3 bags of sweets, each with 10 sweets in. How many sweets do I have altogether?</li> <li>vocalise a multiplication and know that 10 multiplied by 6, for example, means 6 groups of 10</li> <li>represent an abstract multiplication by 10 with a range of equipment and count the total e.g. 4 multiplied by 10</li> <li>produce an array to represent a multiplication</li> <li>recognise a range of words implying 'multiply by 10' and solve problems involving these</li> </ul>	<p>What's the same and what's different? 2 10ps, 10 2ps, <math>2 \times 10</math>, <math>10 \times 2</math></p> <p>What's the same and what's different? Multiplying by 2 and multiplying by 10</p> <p>Convince me that <math>10 \times 6 = 60</math></p>
<p><b>Multiplication by 5</b></p> <ul style="list-style-type: none"> <li>Arranging objects into equal groups and counting efficiently in 5s</li> </ul>	<p>4. Multiply numbers by 5</p> <ul style="list-style-type: none"> <li>count in 5s from 0</li> <li>find the total of objects arranged in tens by counting</li> </ul>	<p>Show me 5 multiplied by 6</p> <p>Always, Sometimes, Never?</p>

<p>(e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on)</p>  <ul style="list-style-type: none"> <li>Using Numicon 5s (or unifix cubes or other attached materials) to represent equal groups of 5 and counting the total by counting efficiently in 5s</li> </ul>  <ul style="list-style-type: none"> <li>Saying the multiplication that has been made. For example, for <math>5 \times 4</math> we would expect to see groups of 5 shown four times and children to practise reading and saying the multiplication as they are building it '4 lots of 5' and '4 groups of 5'.</li> <li>Using an array (with help) to structure the groups of 5 more formally e.g. <math>5 \times 3</math></li> </ul>  <ul style="list-style-type: none"> <li>Using Cuisenaire rods (or straws) to scaling an amount/length by making it five times bigger and so on.</li> <li>Using 5p coins to represent 5s particularly and encourage counting in these denominations</li> </ul>	<p>in 5s (e.g. 5ps, Numicon 5s)</p> <ul style="list-style-type: none"> <li>arrange objects into groups of 5 to count the total</li> <li>given one set of objects, find the numbers in 5 sets by building a four more sets and counting the total</li> <li>given a word problem, represent it with objects and count the total e.g. I have 4 groups of 5 counters. How many counters do I have altogether?</li> <li>vocalise a multiplication and know that 5 multiplied by 6, for example, means 6 groups of 5</li> <li>represent an abstract multiplication by 5 with a range of equipment and count the total e.g. 3 multiplied by 5</li> <li>produce an array to represent a multiplication</li> <li>recognise a range of words implying 'multiply by 5' and solve problems involving these</li> </ul>	<p>5 groups of 2 is the same amount as 2 groups of 5.</p> <p>Convince me the array for <math>5 \times 1</math> is just a line</p> <p>Convince me that <math>5 \times 3 = 15</math></p>
<p><b>Multiplying in general</b></p> <ul style="list-style-type: none"> <li>Using groups of objects set out in hoops (no pattern)</li> <li>Using arrays to represent <math>a \times b</math> as <math>b</math> rows of <math>a</math> counters each to show groups more clearly</li> </ul>	<p>5. Multiply other numbers</p> <ul style="list-style-type: none"> <li>represent a word problem with objects and count them to find the solution</li> <li>represent a given multiplication using a range of equipment</li> <li>recognise a range of words implying multiply and solve problems involving these (with small numbers)</li> </ul>	<p>Show me how you could represent <math>3 \times 6</math></p> <p>Convince me that <math>3 \times 8 = 24</math></p>

	<p>6. Recognise and solve multiplication problems</p> <ul style="list-style-type: none"> <li>• solve problems involving the above as a mixture</li> <li>• self-vocalise a problem and decide how to represent it</li> <li>• say what multiplication a representation shows</li> <li>• record a formal array for multiplication problems with support. The array <math>a \times b</math> should be represented as a columns by <math>b</math> rows (i.e. <math>a</math> across and <math>b</math> down)</li> </ul>	<p>What's the same and what's different? <math>2 \times 6</math>, <math>5 \times 2</math>, <math>10 \times 1</math>, <math>3 \times 4</math></p> <p>What's the same and what's different? 2 groups of 10, 10 groups of 2, 2 numicon 10s, 10 numicon 2s</p>
Further Extension		Rich and Sophisticated Tasks
<p>1. Toy aeroplanes have 5 wheels.  How many wheels would you need to make different numbers of aeroplanes?</p> <p>2.  Lollies cost 5p each. A pack of 3 lollies costs 13p.  How much money do you save when you buy a pack of 3 lollies instead of 3 single lollies?</p> <p>3. Using only 2p, 5p and 10p coins, can you show 20p?  In how many different ways can you do this?  Are you sure you have got them all?  Explain how you know.</p> <p>4. If I start on 0 and count on in fives will I say the number 55?  If I start on 4 and count on in twos will I say the number 17?  If I start at 10 and count on in tens will I say 100?</p>	<p><b>Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</b></p> <p>NRICH: <a href="#">Lots of Biscuits!</a> * P NRICH: <a href="#">Doubling Fives</a> * I</p> <p>NRICH: Using arrays <a href="http://nrich.maths.org/2466">http://nrich.maths.org/2466</a></p> <p>Arrays PPT: <a href="http://topicbox.net/mathematics/multiplication/5112/">http://topicbox.net/mathematics/multiplication/5112/</a></p>	

Misconceptions	Teacher Guidance and Notes
<p>Children may not ensure that all their groups have the same amount when representing a multiplication.</p> <p>Children sometimes struggle to interpret a problem and find the key numbers before deciding what do with them. They cannot unpick the clues within the problem.</p> <p>Children may find it hard to read a formal multiplication e.g. <math>2 \times 5</math> correctly - note that in stage 1 it is acceptable to stick to more informal language e.g. groups of</p> <p>Children tend to use the 'lots of' representation [repeated addition] of multiplication much more than scaling.</p> <p>Children tend to stick to their favourite representations - they may overly rely on, for example, Numicon when a bead string or an array could be more helpful.</p>	<ul style="list-style-type: none"><li>• This is children's first introduction to multiplication although they may have done some repeated addition in earlier units with an adding focus.</li><li>• The intention is for them to become confident at using equipment to represent a problem and then for representing a (verbal) multiplication statement.</li><li>• Division, whilst clearly related to multiplication, is covered explicitly later in Unit 13 so focus here on multiplication.</li><li>• Children must use and encounter the language of multiplication. There is not an expectation that the children themselves will use the notation for multiplications.</li><li>• It is particularly important that children recognise all the different ways of implying a multiplication through language, including multiplied by, times, groups of, lots of, sets of, doubled, twice as big as, and so on. They should also start using this language to orally explain what they have made and what it shows.</li><li>• Use your earlier and continued work on counting in 2s, 5s and 10s to support children in counting efficiently with groups of objects or the rows of an array.</li><li>• Provide the children with plenty of opportunities to use different representations to show the calculation.</li><li>• Encourage the children to organise their groups in a row, which will support them when they move onto more formal arrays. In Stage 1, the expectation that children will use arrays with support but that they will independently produce groups of objects to represent multiplication.</li><li>• Ensure that children are exposed to examples of scaling as well as the more common 'lots of' interpretation of multiplication.</li></ul>
Key Assessment Checklist	
<ol style="list-style-type: none"><li>1. I can set out objects in groups using hoops or support frames to represent a multiplication.</li><li>2. I can say how many there are in the groups I have set out to solve a multiplication. I can check there are the same amount and it is fair.</li><li>3. I can solve a problem involving multiplying by choosing my own objects, arranging them and counting the total.</li><li>4. I can solve a problem involving multiplying by drawing a picture first.</li><li>5. With help I can organise my drawings and objects to represent arrays.</li><li>6. I can represent a multiplication on its own with different equipment and pictures.</li></ol>	

Stage 1	Unit 10 : Investigating Statistics	
<p><b>4 learning hours</b></p>	<p>In this unit children and students explore the collection, representation, analysis and interpretation of data. It covers a range of calculations of central tendency and spread as well as multiple charts and graphs to represent data. As it is the only unit directly exploring statistics, it is critical that children have time to explore the handling data cycle here and to focus sufficient time on interpreting their results.</p>	
Prior Learning	Core Learning	Learning Leads to...
	<p>➤ <b>measure and begin to record the following:</b></p> <ul style="list-style-type: none"> <li>- lengths and heights</li> <li>- mass/weight</li> <li>- capacity and volume</li> <li>- time (hours, minutes, seconds)</li> </ul>	<ul style="list-style-type: none"> <li>➤ interpret and construct simple pictograms, tally charts, block diagrams and simple tables</li> <li>➤ ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity</li> <li>➤ ask and answer questions about totalling and comparing categorical data</li> </ul>
Exemplification		Vocabulary
<p><b>LENGTH</b></p> <p>1. Which line is longer? Explain your reasoning.</p>  <p><b>MASS</b></p> <p>2. Here are three items. Can you sort them from lightest to heaviest by feeling them with your hands?</p> <p><i>Give pupils three items that are quite different in mass.</i></p> <p>3. Which is heavier, a toy car or a toy dinosaur?</p>  <p>4. Which toy is heavier?</p>		<p>object                      longer big                            heavier larger                        lighter small                         ruler tall                            metre rule short                         tape measure long                          scales heavy                         stop watch light                         compare bigger                        centimetres larger                        metres smaller                      grams taller                         kilograms shorter                       seconds                                      minutes</p>



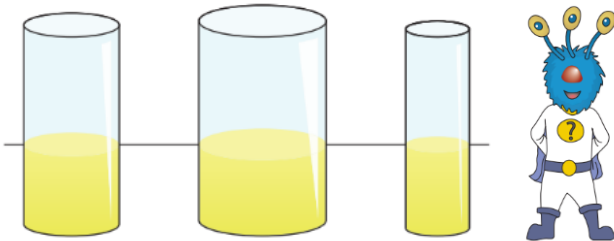
If you added a toy car to the teddy, what would happen to the scales?

Explain your reasoning.

### CAPACITY

5. Captain Conjecture says, 'All of the glasses contain the same quantity of lemonade.'

Do you agree?

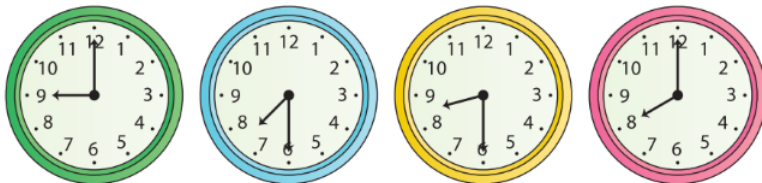


### TIME

6. Sam leaves for school at 8 o'clock. Jay leaves half an hour later than Sam.

Circle the clock which shows when Jay leaves for school.

Explain your reasoning.



7. Circle the times which are shorter than 1 week.

1 year 1 day 1 minute 1 hour 1 month

Representation	Fluency	Probing Questions
<p><b>Length/Height</b></p> <ul style="list-style-type: none"> <li>Get children to make regular estimations and use feedback to improve their subsequent estimations. <a href="http://www.estimate180.com/">http://www.estimate180.com/</a> is a good site to start developing this.</li> <li>Sorting objects into length/height order is a good task</li> <li>To help children choose between cm or m, ask them which they would use to measure certain objects by choosing either a ruler or metre stick</li> </ul>	<p>1. Length/height</p> <ul style="list-style-type: none"> <li>Say if an item is long/tall or short/small</li> <li>Compare two or more items by height/length</li> <li>Choose the correct tool to make a measurement</li> <li>Begin to record lengths</li> </ul>	<p>Show me an object that is taller than a cat</p> <p>Show me an object that could be used to measure the height of this table</p> <p>Convince me that this object is taller than this one</p> <p>What's the same and what's different? (Show 4 objects) eg Pencil, book, cup, bag</p>
<p><b>Mass</b></p> <ul style="list-style-type: none"> <li>Get children to use measuring scales to make predictions of which item is heavier using hands, then confirming with balance scales</li> <li>Lead on to making predictions and then measuring each item individually and comparing bigger/smaller mass</li> <li>Children can use measuring scales to check their mass estimates to develop estimation skills</li> <li>Use measuring scales to find items with mass greater than/less than 100g etc.</li> </ul>	<p>2. Mass</p> <ul style="list-style-type: none"> <li>Compare which item is heavier than another by feeling</li> <li>Use a balance scale to identify greater mass</li> <li>Use a measuring scale to compare two objects</li> <li>Use a measuring scale to identify objects greater/less than a certain mass</li> </ul>	<p>Show me an object that is heavier than a cat</p> <p>Show me an object that could be used to measure the weight of this cup</p> <p>Convince me that this object weighs less than 100g</p>
<p><b>Capacity</b></p> <ul style="list-style-type: none"> <li>Get children to identify which items have greater capacities by pouring in set measurements and counting.</li> </ul>	<p>3. Capacity</p> <ul style="list-style-type: none"> <li>Compare capacities in two similar vessels by sight</li> <li>Compare capacities in two different vessels by pouring and counting given quantities</li> </ul>	<p>What's the same and what's different? Cup; jug; teaspoon; bucket</p> <p>Large containers have a greater capacity than smaller ones</p>
<p><b>Time</b></p> <ul style="list-style-type: none"> <li>Estimate a minute and test who is closest by timing it – get children to put their heads on the desk and shut their eyes before sitting up when they think a minute is up</li> <li>Get children to use stopwatches to test each</li> </ul>	<p>4. Time</p> <ul style="list-style-type: none"> <li>Measure a time using a stop watch</li> <li>Measure a time using a second hand on a clock</li> <li>Compare two different times and say which is longer</li> </ul>	<p>What's the same and what's different? Hours, minutes, seconds</p>

other predicting 5, 10 30 seconds. This will develop estimation skills and use of stopwatch

### Further Extension

1. A long brick is twice the length of a short brick.

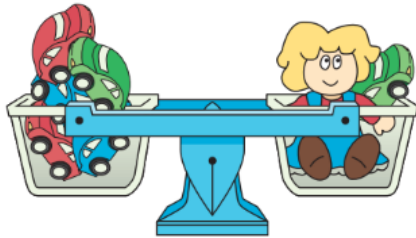
Which is longer:

2 long bricks or 3 short bricks?

3 long bricks or 5 short bricks?



2. Look at these balance scales. There are five cars on one side. The doll weighs the same as how many cars?



3. Point to a glass which is about half as full as the glass in the red oval?

Can you point to a glass which is about twice as full as the glass in the blue oval?

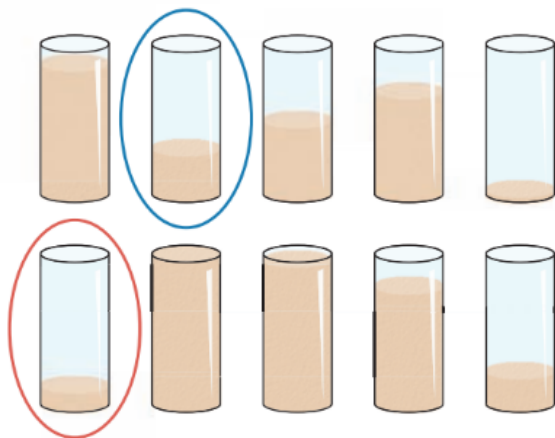
### Rich and Sophisticated Tasks

[Same length trains](#)

[Order, Order!](#)

[Are you thirsty?](#)

The early years activities [available here](#) are a good starting point



### Misconceptions

Children often struggle to bridge their understanding from the word descriptions of measures to a numerical approach. They can articulate whether an object is large or larger but cannot give this a number by measuring.

Children do not always measure an object from 0 - they may not line up an object with the end of a ruler or reset scales before measuring.

When measuring a longer length, children do not always realign the ruler correctly each time.

Children get confused between the objects that are used to measure and the units of measurement that these make use of.

Children do not always realise that units can be formal or informal e.g. length can be measured in handspans or in cm - they do not always include units at all in their answers.

There is a tendency for children to consider capacity to be a measure of liquid rather than of the space inside an object.

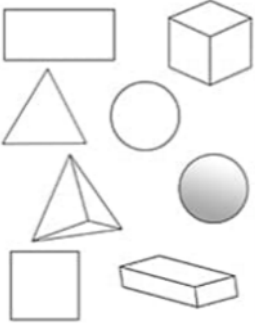
### Teacher Guidance and Notes


- There is a need to be very practical with this unit which focuses on both the skills of measuring and of recording the results.
- Teachers need to bridge the gap between the word descriptions of measures and the allocation of numeric values to measurement. There is a need to model different units.
- Make sure children have opportunities to select their own equipment for measuring and to then reflect on what the units should be and whether this was a sensible choice.
- Equipment needs to be used correctly and this should be modelled with particular focus on measuring from 0, reading accurately and recording carefully.
- When recording measurements, children should include the units and practise doing this using a list or tabular format if possible. Where the units do not match, the teacher needs to draw children's attention to this to make them consider how to address this.

### Key Assessment Checklist

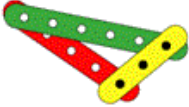

1. I can say if an object is big or small or tall or short or long or short or heavy or light.
2. I can say which object is bigger/taller/longer/heavier (or smaller/shorter/lighter)

3. I can make a prediction about a length or weight or capacity
4. I can record and measure a length by positioning a ruler correctly and reading the amount
5. I can record and measure a weight by using scales and reading the amount.
6. I can record and measure a capacity by repeatedly filling the object and counting this.
7. I can measure and record a time in seconds or minutes using a stop watch.
8. I can decide which object to choose to measure something.

Stage 1	Unit 11 : Visualising Shape																																			
<p><b>8 learning hours</b></p>	<p>In this unit children focus on exploring shapes practically and visually. There is an emphasis on sketching, constructing and modelling to gain a deeper understanding of the properties of shapes. It is therefore necessary to secure the practical skills at the same time as using them to explore the shapes in questions.</p> <p>At secondary level students are developing their skills in construction and the language/notation of shape up to the understanding, use and proof of circle theorems.</p>																																			
Prior Learning	Core Learning	Learning Leads to...																																		
<p>DM: 40-60+ months</p> <ul style="list-style-type: none"> <li>➤ Beginning to use mathematical names for 'solid' 3D shapes and 'flat' 2D shapes and mathematical terms to describe shapes</li> <li>➤ Selects a particular named shape</li> </ul> <p>From ELG: They [children] explore characteristics of everyday objects and shapes and use mathematical language to describe them.</p>	<ul style="list-style-type: none"> <li>➤ <b>recognise and name common 2-D and 3-D shapes, including:</b> <ul style="list-style-type: none"> <li>- <b>2-D shapes [for example, rectangles (including squares), circles and triangles]</b></li> <li>- <b>3-D shapes [for example, cuboids (including cubes), pyramids and spheres]</b></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>➤ identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line</li> <li>➤ identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces</li> <li>➤ compare and sort common 2-D and 3-D shapes and everyday objects</li> </ul>																																		
Exemplification		Vocabulary																																		
<p>1. a) Name these shapes</p>  <p>b) I am thinking of a 3 dimensional shape. It has six faces. What could my shape be?</p>		<table border="0"> <tr> <td>shape</td> <td>sphere</td> </tr> <tr> <td>2D</td> <td>pyramid</td> </tr> <tr> <td>3D</td> <td>cone</td> </tr> <tr> <td>flat</td> <td>corners vertices</td> </tr> <tr> <td>solid</td> <td>edges sides</td> </tr> <tr> <td>circle</td> <td>faces</td> </tr> <tr> <td>square</td> <td>round</td> </tr> <tr> <td>triangle</td> <td>curved</td> </tr> <tr> <td>rectangle</td> <td>smooth</td> </tr> <tr> <td>pentagon</td> <td>straight</td> </tr> <tr> <td>hexagon</td> <td>roll</td> </tr> <tr> <td>octagon</td> <td>sort</td> </tr> <tr> <td>(oval)</td> <td>find</td> </tr> <tr> <td>(semicircle)</td> <td>draw</td> </tr> <tr> <td>cube</td> <td>name</td> </tr> <tr> <td>cuboid</td> <td>equal same</td> </tr> <tr> <td>cylinder</td> <td></td> </tr> </table>	shape	sphere	2D	pyramid	3D	cone	flat	corners vertices	solid	edges sides	circle	faces	square	round	triangle	curved	rectangle	smooth	pentagon	straight	hexagon	roll	octagon	sort	(oval)	find	(semicircle)	draw	cube	name	cuboid	equal same	cylinder	
shape	sphere																																			
2D	pyramid																																			
3D	cone																																			
flat	corners vertices																																			
solid	edges sides																																			
circle	faces																																			
square	round																																			
triangle	curved																																			
rectangle	smooth																																			
pentagon	straight																																			
hexagon	roll																																			
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(semicircle)	draw																																			
cube	name																																			
cuboid	equal same																																			
cylinder																																				

Representation	Fluency	Probing Questions
<p><b>Exploring 2D Shapes</b></p> <ul style="list-style-type: none"> <li>Using shapes to make patterns and describing these</li> <li>Using shapes to make pictures – mosaic pieces, sticky paper shapes, ‘fuzzy felts’. Can they make an animal with a circular head, a triangular body etc... Ask the children to make ‘ruler’ pictures –use a ruler to draw a series of intersecting lines and to colour in each enclosed shape, talking about the properties of these shapes and naming them as they do so.</li> <li>Playing with tangrams to explore the properties of a square. <b>Nrich</b> version</li> <li>Going on a shape hunt to find specific shapes in real life e.g. rectangles</li> <li>Counting (and marking off) the vertices (edges/faces) of a 3D or a 2D shape</li> <li>Drawing round 2D shapes</li> <li>Using the <a href="#">Polygon ITP</a> to explore shapes with ICT</li> </ul>	<p>1. Describe and explore the properties of 2D shapes</p> <ul style="list-style-type: none"> <li>say if the sides are curved or straight</li> <li>say if there are corners</li> <li>say whether it will roll</li> <li>say whether it can be folded in half?</li> <li>begin to count the sides and vertices</li> </ul> <p>2. Use some mathematical language to describe 2D shapes</p> <ul style="list-style-type: none"> <li>name the shape</li> <li>state number of sides</li> <li>state number of vertices</li> <li>begin to say whether any of the sides are equal in/same length</li> </ul>	<p>Show me a shape that will roll</p> <p>Show me a shape with only straight edges</p> <p>Show me a shape that you don't know the name of....</p> <p>Convince me that this shape has 6 vertices</p> <p>What's the same and what's different? triangle and square</p> <p>Always, Sometimes, Never? Triangles have three sides</p>
<p><b>Exploring 3D Shapes</b></p> <ul style="list-style-type: none"> <li>Exploring the feel of 3D (and 2D) shapes to sense whether they are made of rounded faces or flat faces, for example, using a ‘feely bag’</li> <li>Exploring and handling 3D shapes using mathematical models as well as everyday objects, especially packaging!</li> </ul>  <ul style="list-style-type: none"> <li>Making models of shapes using plasticine or</li> </ul>	<p>3. Describe the properties of 3D shapes</p> <ul style="list-style-type: none"> <li>say if the faces are curved (round) or flat</li> <li>say if the edges/sides are curved or straight (or both e.g. cone)</li> <li>say whether it will roll</li> <li>say whether there are corners</li> <li>describe the shape of the faces</li> <li>begin to count the faces and vertices (and possibly edges)</li> </ul> <p>4. Use some mathematical language to describe 3D shapes</p> <ul style="list-style-type: none"> <li>name the shape</li> <li>state the number of faces</li> <li>state the number of vertices</li> <li>state the number of edges</li> <li>state the shapes that make up the faces</li> </ul>	<p>Show me a shape that will roll</p> <p>Show me a shape that can be stacked</p> <p>Show me a shape that you don't know the name of....</p> <p>Convince me that this is a 3D shape</p> <p>Convince me that a cube has 6 faces</p> <p>True or False? A pyramid has more faces than edges</p>

<p>using construction materials (e.g. blocks, duplo, multi-link etc)</p> <ul style="list-style-type: none"> <li>• Exploring making and ‘unmaking’ 3D shapes using nets/cereal packets etc [can be linked to D&amp;T or other activities e.g. making a box]</li> <li>• Printing with 3D shapes to explore the shapes of the faces. Which shapes have square faces? Did you print with any shapes with circular faces? What happens when you print with a sphere? A cylinder?</li> </ul>	<ul style="list-style-type: none"> <li>○ begin to say whether any of or all of the faces and/or edges are the same or equal</li> </ul>	
<p><b>Naming Shapes in different orientations</b></p> <ul style="list-style-type: none"> <li>• Drawing shapes in sand or with chalk outside, then moving to stand and look at these in different positions</li> <li>• Making 2D shapes out of sticks (or children!)</li> <li>• Folding paper in half in different ways. If they make one fold on a square, what shapes can they make? Can they make one fold and make a four-sided shape that isn’t a rectangle?</li> </ul>	<p>5. Name 2D and 3D shapes in different orientations</p> <ul style="list-style-type: none"> <li>○ name 2D shapes with base parallel to the bottom of the page/table</li> <li>○ name 3D shapes with base parallel to bottom of page/table</li> <li>○ name 2D shapes when rotated</li> <li>○ name 3D shapes when rotated</li> <li>○ name shapes involved in a combination/compound shape</li> <li>○ name the shape made by putting two known shapes together</li> </ul>	<p>Convince me that this is definitely a cube</p> <p>Always, Sometimes, Never? 2D shapes have 3 sides</p> <p>Convince me that this is a circle</p>
<p><b>Sorting</b></p> <ul style="list-style-type: none"> <li>• Sorting shapes into categories e.g. find all the triangles here</li> <li>• Using hoops to make a Venn diagram to sort shapes out into groups</li> </ul>	<p>6. Sort 2D and 3D shapes into given categories</p> <ul style="list-style-type: none"> <li>○ categories by name e.g. triangles or cubes</li> <li>○ categories by properties e.g. 3D shapes or shapes with 4 vertices or shapes with all straight sides</li> </ul>	<p>Show me how you could sort these shapes into 2 groups - what rule did you use?</p> <p>Show me a shape beginning with c</p> <p>What’s the same and what’s different? cylinder, cone, circle, sphere</p> <p>What’s the same and what’s different? square and rectangle</p>
<p><b>Naming shapes from descriptions</b></p>	<p>7. Given a description of a shape, name the shape</p>	<p>Always, Sometimes, Never?</p>

<ul style="list-style-type: none"> <li>Finding all the shapes that match a given rule e.g. a shape with 6 vertices or a shape with all sides the same length</li> <li>Playing peekaboo with shapes, trying to guess the shape as more and more is revealed</li> <li>Playing 'Guess the Shape' from a description or using a 20 questions format (ensure children get to 'be the teacher' here too)</li> <li>Using plastic geostrips to investigate triangles with sides of different lengths</li> </ul> 	<ul style="list-style-type: none"> <li>situations with only one right answer</li> <li>situations where multiple shapes meet the description</li> </ul>	<p>Shapes starting with c can roll</p> <p>Always, Sometimes, Never? A shape with 4 vertices must be 2D</p>
<p><b>Describing shape from the name</b></p> <ul style="list-style-type: none"> <li>Matching the names of shapes to images of the shapes <a href="#">NRich version of shape cards here</a></li> </ul>	<p>8. Given the name of shape, find an example of it and describe its main features</p> <ul style="list-style-type: none"> <li>rectangle, square, triangle, circle</li> <li>pentagon, hexagon, octagon, oval, semicircle etc</li> <li>cube, cuboid, sphere, pyramid</li> <li>cone, cylinder, .... -based pyramid, prism, hemisphere etc.</li> </ul>	<p>Show me what is special about a triangle</p> <p>What's the same and what's different? triangle and circle</p> <p>What's the same and what's different? cube and cuboid</p>
<p><b>Further Extension</b></p>		<p><b>Rich and Sophisticated Tasks</b></p>
<p>1. (Repeated from Exploring Shape Unit)</p> <p>What's the same and what's different about these shapes?</p>  <p>Which could be the odd one out and why?</p> <p>Could each one be the odd one out?</p> <p>Explain your reasoning.</p>		<p><b>NRICH:</b> Building with Solid Shapes (<a href="http://nrich.maths.org/239">http://nrich.maths.org/239</a>)</p> <p><b>NRICH:</b> Logic Block Collections (<a href="http://nrich.maths.org/6032">http://nrich.maths.org/6032</a>)</p> <p><b>NRICH:</b> Sorting Shapes (<a href="http://nrich.maths.org/5997">http://nrich.maths.org/5997</a>)</p> <p><b>Tasks Repeated from Exploring Shape Unit</b></p> <p><b>NRICH:</b> <a href="#">Shaping It</a> * I</p> <p><b>NRICH:</b> <a href="#">What's Happening?</a> * P</p> <p><b>NRICH:</b> <a href="#">Jig Shapes</a> * P</p> <p><b>NRICH:</b> <a href="#">Overlaps</a> ** P</p>

## 2. (Repeated from Exploring Shape Unit)

Tom says, 'My shape has 4 rectangular faces and 2 square faces. What is my shape?'

Sam says, 'My shape has 2 triangular faces and 3 rectangular faces. How many vertices does my shape have?'

### Misconceptions

The most common misconception is a reliance on orientation to identify a shape and hence the failure to recognise a square when it is rotated (as in the exemplification above).

Children may confuse flat and solid shapes and fail to see the difference between the two types. This is exacerbated by too much exposure to images of 3D shapes rather than the objects themselves.

Children may want to call curved edges- circle edges. They may also find it hard to work with shapes with both curved and straight edges e.g. semicircle or cone

Children may confuse a cube with a square because of the relationship between these- language reinforcement from the start using faces, edges, vertices etc.

Squares and rectangles can be confused - bear in mind that later we want children to see a square as a 'special case' of a rectangle

### Teacher Guidance and Notes

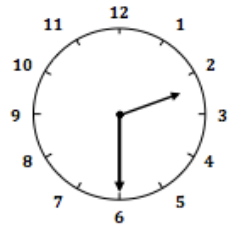
- The content of this unit is the same as that of the earlier unit, Exploring Shape and hence there is some element of reviewing and revisiting previous learning here.
- However, the expectation is that by this stage children are able to be more mathematical in their descriptions of shapes and to know a wider range of both 2D and 3D shapes.
- You may find that children are more able to move on to later mastery stages of the model in this unit than previously.
- Children should be exposed to the mathematical language as much as possible, although they may not mirror this back at first. For example, use language such as equal, vertices, edges etc. wherever possible
- There is no definitive list of shapes that must be known at this level (although all those in the objective are essential). Therefore, expect questioning to cover common 2D and 3D shapes as listed in the vocabulary box above.
- It is very important that you vary the orientation of the shapes that you show children so that they do not implicitly 'presume' that the shape has to be a certain way up to qualify as a triangle, for example.
- During this unit constantly reinforce the properties of these shapes, as this is what will define mastery at this stage, rather than simply the ability to name the shapes.
- Don't be frightened to introduce other shapes and their technical names - it is better that children use this vocabulary from the start e.g. rhombus rather than diamond and hemisphere rather than half a sphere. Young children generally relish learning new words and will be keen to expand their vocab and impress their parents with their knowledge of cylinders, kites and so on.

### Key Assessment Checklist

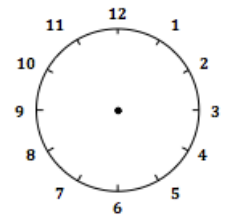
1. I can recognise and name rectangles, squares, triangles and circles (in environment and on paper)
2. I can sort and group a range of 2D shapes
3. I can describe the properties of a 2D shape
4. I can name a 2D shape when someone describes it
5. I can recognise and name cubes, cuboids, pyramids and spheres (in environment and on paper)
6. I can sort and group a range of 3D shapes
7. I can describe the properties of a 3D shape
8. I can name a 3D shape when someone describes it

Stage 1	Unit 12 : Exploring Change			
<p><b>8 learning hours</b></p>	<p>For primary pupils this unit focuses on the measures elements of time and co-ordinates. There is a progression from sequencing and ordering through telling the time formally to solving problems involving time. The co-ordinate work flows in the secondary students' learning focused on the relationships between co-ordinates. Key objectives include the use of <math>y=mx+c</math> for straight lines, the use of functions and the graphing of more complex functions.</p>			
Prior Learning	Core Learning	Learning Leads to...		
<p>DM 22-36 Months:</p> <ul style="list-style-type: none"> <li>&gt; Understands some talk about the immediate past or future e.g. before, later, soon</li> <li>&gt; Anticipates specific time-based events such as meal times or home time</li> </ul> <p>DM 40-60 Months:</p> <ul style="list-style-type: none"> <li>&gt; Uses everyday language related to time</li> <li>&gt; Orders and sequences familiar events</li> <li>&gt; Measures short periods of time in simple ways</li> </ul> <p>ELG:</p> <ul style="list-style-type: none"> <li>&gt; Children use everyday language to talk about time, to compare quantities and to solve problems.</li> </ul>	<ul style="list-style-type: none"> <li>&gt; <b>recognise and use language relating to dates, including days of the week, weeks, months and years</b></li> <li>&gt; <b>tell the time to the hour and half past the hour and draw the hands on a clock face to show these times</b></li> <li>&gt; <b>sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening]</b></li> </ul>	<ul style="list-style-type: none"> <li>&gt; compare and sequence intervals of time</li> <li>&gt; tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times</li> <li>&gt; know the number of minutes in an hour and the number of hours in a day.</li> </ul>		
Exemplification		Vocabulary		
<p>1. What comes next in each sequence?</p> <p>a) Tuesday, Wednesday, Thursday, .....</p> <p>b) January, February, March, .....</p> <p>c) What is today's date?</p>		<p>before after next first then later today yesterday tomorrow</p>	<p>Monday Tuesday Wednesday Thursday Friday Saturday Sunday month January</p>	<p>August September October November December season spring summer autumn</p>

2. a) What is the time?



b) Show the time **8 o'clock** on this clock



3.

a) Put these steps for a birthday cake in the right order:

Cut the cake	Eat the cake	Light the candles	Sing Happy Birthday	Blow out the candles
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Explain your order.

b) Name something you do before lunch/after lunch/between play and lunch

morning afternoon evening days of the week	February March April May June July	winter day week weekend year
--	---	--

Representation	Fluency	Probing Questions
<p><b>Vocabulary of Time</b></p> <ul style="list-style-type: none"> <li>Exploring calendars (paper and electronic) to find names of days, months and number of days in each month</li> <li>Completing/updating the date every morning including the day of the week, date and month</li> <li>Using and completing a daily schedule/plan each day to understand the events and their order in the school day.</li> <li>Producing own daily schedule for the weekend.</li> </ul>	<p>1. Know and use the days of the week</p> <ul style="list-style-type: none"> <li>read and write the names of each day</li> <li>order the days of the week</li> <li>say which day comes next/is tomorrow</li> <li>say which day came before/was yesterday</li> </ul>	<p>Show me a weekend day Show me a day of the week when you would be at school</p> <p>Show me the day tomorrow Show me the day yesterday</p> <p>Convince me that there are 7 days in a week</p> <p>What's the same and what's different? Monday; Tuesday; Friday; Saturday</p>

<ul style="list-style-type: none"> <li>• Chanting days of the week and months and associating with actions involving a timeline (counting stick/number track) or circle</li> <li>• Sequencing days of the week and months on cards</li> <li>• Sorting months by the number of days in them</li> </ul>		<p>Always, Sometimes, Never? ... Saturday is at the weekend</p>
	<p>2. Know and use the months of the year</p> <ul style="list-style-type: none"> <li>• read and write the names of each month</li> <li>• name all four seasons</li> <li>• order the seasons</li> <li>• say which months are in which season</li> <li>• order the months of the year</li> <li>• say which month comes next</li> <li>• say which month came before</li> <li>• ext: begin to recognise how many days are in each month (and that they are not all the same)</li> </ul>	<p>Show me the month that comes after April Show me a month in winter</p> <p>Convince me that a month is longer than a week</p> <p>What's the same and what's different? April, May, June, Friday, Saturday</p> <p>Always, Sometimes, Never? ... months are longer than days</p> <p>Always, Sometimes, Never? ... there are 12 months</p> <p>Always, Sometimes, Never? ... September is a day</p>
	<p>3. Know and use the language of dates</p> <ul style="list-style-type: none"> <li>• read/write a date in words</li> <li>• read a date from a calendar</li> <li>• write the date</li> <li>• say/write important dates e.g. birthday</li> <li>• know that years are given as numbers (now with 4 digits)</li> <li>• say/read the current year and year of birth</li> </ul>	<p>Convince me that you were born after a person in year 6</p> <p>What's the same and what's different? yesterday, today, tomorrow</p>
<p><b>Telling Time</b></p> <ul style="list-style-type: none"> <li>• Exploring a clock together to see how it works</li> <li>• Relating times to daily routine</li> <li>• Referring the clock in the classroom and labelling this with Numicon as well as word</li> </ul>	<p>4. Tell/show the time to the hour</p> <ul style="list-style-type: none"> <li>• read the time from a clock e.g. 2 o'clock</li> <li>• write a time using 'o'clock'</li> <li>• show a time to the hour on a clock</li> <li>• draw hands on a clock face to show a time to the hour</li> </ul>	<p>Show me a clock showing 2 o'clock</p> <p>Show me the time that this clock is showing</p> <p>Show me a time on the clock when you could have tea</p>

<ul style="list-style-type: none"> <li>labels</li> <li>Making clocks using paper plates, card sticks and split pins for hands</li> <li>Using manipulative clocks to show and read times (preferably mini-clocks for each child and a larger one for the teacher)</li> <li>Making human clocks using arms</li> <li>Counting in 'o'clocks' aloud and linking to a clock image (possibly with an action using arms to show each time)</li> </ul>		<p>Convince me that the time on the clock is five o'clock</p> <p>What's the same and what's different? Hours and days</p>
<p><b>Ordering Events</b></p> <ul style="list-style-type: none"> <li>Role-playing familiar processes first to help order the steps as cards afterwards. For example, buying an item in a shop, borrowing a library book, getting ready in the morning. Then testing out the order by acting the cards out and checking whether this makes sense.</li> <li>'Rewinding' actions to help associate with past tense and words such as before/yesterday etc.</li> <li>"Fast-forwarding" actions to help associate with future tense and words such as later/soon/tomorrow etc.</li> </ul>	<p>5. Tell/show the time to the half hour</p> <ul style="list-style-type: none"> <li>read the time from a clock e.g. half past five</li> <li>recognise that the hour hand lies between the two hours</li> <li>write a time in words</li> <li>show a time to the half hour on a clock</li> <li>draw hands on a clock face to show a time to the half hour</li> </ul> <p>6. Order and sequence short term events</p> <ul style="list-style-type: none"> <li>suggest events in daily routine e.g. things that happen at school</li> <li>order events in daily routine</li> <li>suggest events in familiar short routines (e.g. shopping/eating lunch/birthday cake/getting ready for PE)</li> <li>order events in familiar short routines</li> <li>suggest what is wrong with events that have been incorrectly sequenced (and correct)</li> </ul> <p>7. Order and sequence longer term events (in a week, year etc)</p> <ul style="list-style-type: none"> <li>days of the week</li> <li>months of the year</li> <li>seasons</li> <li>weekly routines e.g. within school (singing, PE, swimming etc)</li> <li>events that happen through the year e.g. Christmas, Easter, your birthday, ....</li> </ul>	<p>Show me a clock showing half past three</p> <p>Show me a time on the clock where you would be in school</p> <p>Show me the time school starts/finishes</p> <p>Convince me that the time on the clock is half past eleven</p> <p>Always, Sometimes, Never? ... you have lunch at 12 o'clock</p> <p>Always, Sometimes, Never? breakfast comes before dinner</p> <p>Show me the month that comes before August</p> <p>Show me the day that comes after Thursday</p> <p>Show me something that happens between Christmas and Easter</p>

8. Solve problems involving time and sequencing
- order times (hours or half past)
  - order dates
  - order people by their birthdays
  - suggest a time or date that lies between two given times/dates
  - suggest an event that lies between two given events
  - suggest the time given just one hand of a clock

Always, Sometimes, Never?  
time is measured in minutes

Always, Sometimes, Never?  
... time can be measured in lengths of a ruler

What's the same and what's different?  
the big hand on the clock and the little hand on the clock

### Further Extension

### Rich and Sophisticated Tasks

1.  
Match the clocks to the following times:



half past nine

five o'clock

half past two

seven o'clock

2.  
Jackie is looking forward to the events marked on the calendar.

January						
Sun	Mon	Tue	Weds	Thurs	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

Use the clues to identify the date that she carried out each activity.

Jackie is going to a party at the weekend. This is  January.

She is visiting her aunty on a Tuesday. This is  January.

Three days after the party she is going swimming. This is  January.

Near the end of the month she is going to the cinema. This is  January.

Sequence events in chronological order using language (for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening)

NRICH: [The Games' Medals](#) \*\* I

NRICH: [Times of Day](#) \* P I

Recognise and use language relating to dates, including days of the week, weeks, months and years

NRICH: [Snap](#) \* G

3.  
Sam leaves for school at 8 o'clock. Jay leaves half an hour later than Sam.  
Circle the clock which shows when Jay leaves for school.

Explain your reasoning.



Circle the times which are shorter than 1 week.

1 year    1 day    1 minute    1 hour    1 month

4.  
I walk to school every day. On Monday my journey takes 10 minutes.  
On Tuesday I walk more slowly. Does my journey take more or less time than on Monday?

Explain your answer.

On Wednesday it takes me 8 minutes to walk to school.  
On which of the 3 days do I walk quickest?  
On which of the 3 days do I walk slowest?



Explain your reasoning.


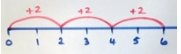

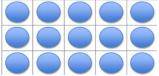
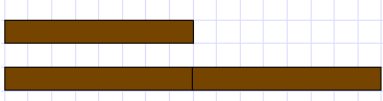
5.  
Here are some clocks where the minute hand has broken off.  
Use the hour hand to work out what time it is.

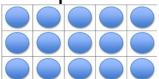


Misconceptions	Teacher Guidance and Notes
<p>Children may struggle with the large volume of information and language involved here - for instance, they may muddle the days of the week or the months of the year.</p> <p>Children may assume that every month has the same number of days.</p> <p>Due to the decimal system, children may assume that there are 10 months in the year.</p> <p>Children will frequently confuse the hands on the clock so from the start identify that the hour hand points towards the hour, while the minute hand is much longer pointing at the minutes. Often, the issue is that children think the big hand should be the hour hand because it is the most important.</p> <p>Children may not always say half <b>past</b> the hour and either miss this out or replace with before, after etc.</p> <p>Sometimes children move the hands of a clock in an anticlockwise direction.</p> <p>Sometimes children order items using a different criterion (to chronological order).</p>	<ul style="list-style-type: none"><li>• This unit is the first formal introduction to time that children have had.</li><li>• However, this topic lends itself to regular, daily reference and practice and so children should have encountered aspects of saying/writing the date, ordering events in daily routine, referring to a clock and so on throughout the classroom experience.</li><li>• It is recommended that you display and use a standard clock in your classroom, labelling this with Numicon and words.</li><li>• It is also recommended that you make the date (day, date, month) and season part of your daily routine.</li><li>• Spend time getting familiar with the facts and vocabulary of time as there is a considerable volume of specialist language.</li><li>• Be prepared for some children to be experts at this topic already and some to have never seen an analogue clock - there will be a big range of experience and prior knowledge so the establishment of starting points process is even more critical here.</li><li>• Note that officially Sunday is the first day of the week (for historic reasons).</li><li>• Note that September, ..., December were originally the 7<sup>th</sup>-10<sup>th</sup> months of the year (hence their names) but this was distorted when two new months were added (January and February).</li><li>• Take the opportunity to link cardinal numbers to items (first, second, ... ) when sequencing. This can be in word or numeral form.</li></ul>
Key Assessment Checklist	
<ol style="list-style-type: none"><li>1. I can use the vocabulary of time and dates (including days of week, months of year)</li><li>2. I can recall key facts about time e.g. how many days of the week there are, the names of the months</li><li>3. I can read the time to the hour</li><li>4. I can show the time to the hour.</li><li>5. I can tell the time to the half hour.</li><li>6. I can show the time to the half hour</li><li>7. I can put everyday events in order.</li><li>8. I can order everyday events (using vocabulary e.g. before, after, next, first)</li></ol>	

Stage 1	Unit 13: Proportional Reasoning																			
<p><b>8 learning hours</b></p>	<p>In this unit pupils explore proportional relationships, from the operations of multiplication and division on to the concepts of ratio, similarity, direct and inverse proportion.</p> <p>For primary pupils in Stages 1-3, this is focused on developing skills of division. Stages 4 and 5 revisit the whole of calculation to broaden to all four operations in a range of contexts and combination problems; the emphasis here is really on representing and then solving a problem using their calculation skills, not just calculating alone.</p> <p>In Stage 6 the real underpinning concepts of proportion and ratio develop.</p> <p>Secondary pupils begin to formalise their thinking about proportion by finding and applying scale factors, dividing quantities in a given ratio and fully investigating quantities in direct or inverse proportion, including graphically.</p>																			
Prior Learning	Core Learning	Learning Leads to...																		
<p>➤ 40-60+ months: Children find the total number of items in two groups by counting all of them.</p> <p>ELG: Children solve problems, including doubling, halving and sharing.</p>	<p>➤ <b>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</b></p>	<p>➤ calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (<math>\times</math>), division (<math>\div</math>) and equals (=) signs</p> <p>➤ show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot</p> <p>➤ solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts</p>																		
Exemplification		Vocabulary																		
<p>1. a) 25 sweets are shared between 5 children. How many sweets does each child receive? b) Represent the calculation 18 divided by 2 with objects c) Draw/make an array to show 15 divided by 5</p>		<table border="0"> <tr> <td>count</td> <td>division</td> </tr> <tr> <td>objects</td> <td>divided by</td> </tr> <tr> <td>share</td> <td>problem</td> </tr> <tr> <td>shared between</td> <td>represent</td> </tr> <tr> <td>shared into</td> <td>how many</td> </tr> <tr> <td>share equally</td> <td>equals</td> </tr> <tr> <td>groups</td> <td>same as</td> </tr> <tr> <td>equal groups</td> <td>fair</td> </tr> <tr> <td>array</td> <td>even</td> </tr> </table>	count	division	objects	divided by	share	problem	shared between	represent	shared into	how many	share equally	equals	groups	same as	equal groups	fair	array	even
count	division																			
objects	divided by																			
share	problem																			
shared between	represent																			
shared into	how many																			
share equally	equals																			
groups	same as																			
equal groups	fair																			
array	even																			

Representation	Fluency	Probing Questions
<p><b>Counting in 2s, 5s, and 10s</b></p> <ul style="list-style-type: none"> <li>Using a counting stick to count up and down in 2s, 5s or 10s</li> <li>Removing some labels to help recall these key numbers</li> </ul>	<p>1. Recap: count in 2s, 5s and 10s</p> <ul style="list-style-type: none"> <li>count in 2s forwards from 0</li> <li>count backwards in 2s</li> <li>count in 5s forwards from 0</li> <li>count backwards in 5s</li> <li>count in 10s forwards from 0</li> <li>count backwards in 10s</li> </ul>	<p>Convince me that if I can count in 5's that I am also able to count in 10's</p> <p>Convince me that if I count in 5's the number will always end in a 5 or a 0</p>
<p><b>Sharing</b></p> <ul style="list-style-type: none"> <li>Sharing objects into equal groups and counting the number in each group (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on) e.g. 6 frogs shared onto 3 lily pads</li> </ul> 	<p>2. Divide numbers by sharing objects</p> <ul style="list-style-type: none"> <li>share a set of objects into 2 groups and count the number in each group</li> <li>share a set of objects into 3 or 4 groups and count the number in each group</li> <li>share a set of objects into 5 or 10 groups and count the number in each group</li> <li>count out the right number of objects and share them into the correct number of groups, counting the number in each group</li> <li>check whether a sharing is fair by counting the number in each group to see if they are equal</li> <li>given a simple word problem, represent it with objects, share them and count the number in each group e.g. I have 12 counters. I share them between 3 children. How many counters does each child get?</li> </ul>	<p>Show me 12 shared into 2 groups Show me 12 shared into groups of 2</p> <p>Show me a number which I can share evenly into two groups</p> <p>Show me a number which can be shared evenly into 5 groups</p> <p>Show me a number which can not be divided evenly into 5 or 2 groups</p> <p>Convince me that 18 shared between 2 is 9</p> <p>Convince me that half of 12 is 6</p>
<p><b>Grouping</b></p> <ul style="list-style-type: none"> <li>Grouping objects and counting the number of groups (e.g. sweets, animals, books, bean bags, counters, cubes and so on arranged into hoops, lily pads, boxes, bags, and so on) e.g. 15 shapes grouped in 5s</li> </ul>  <ul style="list-style-type: none"> <li>Using a bead string to represent groups of 2 (i.e. repeated subtraction) and count the number of groups (This is a useful precursor</li> </ul>	<p>3. Divide numbers by grouping objects</p> <ul style="list-style-type: none"> <li>group a set of objects into 2s (pairs) and count the number of groups</li> <li>group a set of objects into 3s or 4s and count the number of groups</li> <li>group a set of objects into 5s or 10s and count the number of groups</li> <li>count out the right number of objects and group them correctly, counting the number of groups produced</li> <li>check whether a grouping is fair by checking that all the groups are of equal size</li> <li>given a simple word problem, represent it with</li> </ul>	<p>Show me the array for 8 shared into groups of 2</p> <p>What's the same and what's different? sharing; grouping</p>

<p>to the number line) e.g. 6 divided by 2 (grouped into 2s)</p>  <ul style="list-style-type: none"> <li>Using a number line for repeated subtraction to count back in 2s, 5s or 10s.</li> </ul>  <ul style="list-style-type: none"> <li>Pegging socks in pairs on to a washing line to divide by 2 e.g. 8 divided by 2 (4 pairs)</li> </ul> 	<p>objects, group them and count the number of groups. e.g. I have 15 sweets. I share them into bags of 5 sweets. How many bags can I make?</p>	
<p><b>Arrays</b></p> <ul style="list-style-type: none"> <li>Exploring real life arrays such as egg boxes, cake trays and chocolate boxes</li> <li>Using an array (with help) to structure the groups more formally e.g. 15 divided by 3</li> </ul> 	<p>4. Produce an array to represent a division</p> <ul style="list-style-type: none"> <li>produce an array to represent a grouping problem e.g. 12 grouped in 2s</li> <li>know that the number of groups is the number of items in each row of the array</li> <li>produce an array to represent a simple word problem e.g. There are 20 children in the class. They are put in groups of 5. How many groups of children are there?</li> </ul>	<p>Show me the array that represents 16 divided by 2</p> <p>Show me an array that can be made out of 20 counters</p> <p>Convince me the array for 10 divided by 1 is just a line</p>
<p><b>Other representations</b></p> <ul style="list-style-type: none"> <li>Drawing sharing by using dots in circles</li> <li>Drawing grouping by drawing an array</li> <li>Using Cuisenaire rods (or straws) to scaling an amount/length by making it half as big/making it ten times smaller and so on. For example, halving 16</li> </ul> 	<p>5. Divide numbers using visual representations</p> <ul style="list-style-type: none"> <li>group drawn objects into 2s, 3s and so on, before counting the number of groups</li> <li>share drawn objects using arrows, before counting the number in each group</li> <li>draw an array to represent a division, counting the number of columns (or the number in each row) to find the number of groups</li> </ul>	<p>What's the same and what's different? <math>12 \div 2</math>; <math>20 \div 5</math>; <math>40 \div 10</math>; <math>30 \div 5</math></p> <p>Always, Sometimes, Never 15 divided by 3 = 30 divided by 10</p>
<p><b>Recognising Division Problems</b></p> <ul style="list-style-type: none"> <li>Exploring sharing and grouping situations to</li> </ul>	<p>6. Recognise division problems</p> <ul style="list-style-type: none"> <li>vocalise a division and know that 30 divided by 10</li> </ul>	<p>Show me the division that this represents</p>

<ul style="list-style-type: none"> <li>look for words that suggest a division</li> <li>Saying the division that has been made. For example, reading the division from an array or from a grouping.</li> </ul>	<p>means 30 shared into 10 groups OR 30 grouped into 10s.</p> <ul style="list-style-type: none"> <li>represent an abstract division with a range of equipment and find the answer e.g. 14 divided by 2</li> <li>recognise a range of words implying 'divide by ...' and solve problems involving these</li> <li>say which division a representation (including an array) represents</li> </ul>	<p>What's the same and what's different? An array for <math>10 \div 2</math> and an array for <math>10 \div 5</math></p> <p>Always, Sometimes, Never? 6 divided by 2 equals 4</p>
<p><b>Multiplication</b></p> <ul style="list-style-type: none"> <li>Representing multiplications as groups e.g. <math>5 \times 3</math> as three groups of 5 objects</li> <li>Building and drawing arrays to represent multiplication e.g. <math>5 \times 3</math></li> </ul> 	<p>7. Recap multiplication processes</p> <ul style="list-style-type: none"> <li>represent a multiplication problem using groups</li> <li>represent a multiplication problem using an array</li> <li>solve a simple word multiplication problem</li> <li>recognise a range of words implying 'multiply by ...' and solve problems involving these</li> </ul>	<p>What's the same and what's different? 2 10ps, 10 2ps, <math>2 \times 10</math>, <math>10 \times 2</math>, 2 groups of 10, 10 groups of 2, 2 numicon 10s, 10 numicon 2s</p>
<p><b>Representing problems</b></p> <ul style="list-style-type: none"> <li>Building an object version of a problem to help decide whether to multiply or divide and to find the solution</li> </ul>	<p>8. Recognise and solve multiplication and division problems</p> <ul style="list-style-type: none"> <li>represent a word problem with objects and count the total/number of groups to find the solution</li> <li>represent a given multiplication or division using a range of equipment</li> <li>say which multiplication and division an array represents</li> <li>recognise a range of words implying multiply and divide and solve problems involving these (with small numbers)</li> <li>solve problems involving the above as a mixture</li> </ul>	<p>What's the same and what's different? <math>30</math> divided by 2, 5 multiplied by 3, <math>30</math> divided by 5, <math>30</math> divided by 10; 3 multiplied by 10</p> <p>Always, Sometimes, Never Dividing by 2 gives you an even answer</p> <p>Always, Sometimes, Never You can only halve even numbers</p>
<b>Further Extension</b>		<b>Rich and Sophisticated Tasks</b>
<p>1. How else could 20 sweets be put into bags so that every bag had the same number of sweets?</p> <p>How many bags would be packed each time?</p> <p>2.</p>		<p>Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher</p> <p><b>NRICH:</b> <a href="#">Share Bears</a> * G</p>

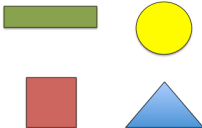
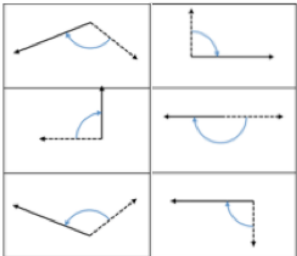
<p>I can see 10 wheels. How many bicycles?</p> <p>3. Lollies cost 5p each. A pack of 3 lollies costs 13p.</p> <p>How much money do you save when you buy a pack of 3 lollies instead of 3 single lollies?</p> <p>4.</p> <p>Using only 2p, 5p and 10p coins, can you show 20p?</p> <p>In how many different ways can you do this?</p> <p>Are you sure you have got them all?</p> <p>Explain how you know.</p>	
<b>Misconceptions</b>	<b>Teacher Guidance and Notes</b>
<p>Children sometimes struggle to interpret a problem and find the key numbers before deciding what do with them. They cannot unpick the clues within the problem.</p> <p>Children tend to stick to their favourite representations - they may overly rely on, for example, groups of objects in hoops when a bead string or an array could be more helpful.</p> <p>Children confuse the processes of sharing (into a given number of piles - like dealing cards - and seeing how many objects are in each pile at the end) and grouping (counting out groups of a given number and seeing how many groups you end up with). Note that they may also tend to allow one to dominate and therefore not gain much practice with the other.</p> <p>Children will not always naturally notice the connection between multiplication and division - they need you to draw these properties out to make the links clearer.</p> <p>Similarly children may not notice the connection between fractions and division.</p> <p>Children may not realise that all numbers can be divided by 2, 5 and 10, but not all numbers will divide exactly (with no objects left over).</p>	<ul style="list-style-type: none"><li>• This unit builds on the multiplication work of Unit 9. It is intended that the representations and images are similar to help children link the two processes.</li><li>• In Stage 1 there is an expectation that children will use and encounter the language of division, but not that they will themselves use the notation e.g. you might expect them to say shared between rather than divided by and to only begin to recognise <math>12 \div 2</math></li><li>• Ensure time to cover both processes of sharing and grouping (as there can be a tendency to focus more on sharing).</li><li>• When progressing from grouping in hoops to arrays, encourage children to 'organise' their groups so that you can see straight away if there are any extra or missing objects. This will lead you nicely on to the array as way of neatly showing your groups (each group is a column).</li><li>• Earlier and regular work on counting in 2s, 5s and 10s should be rehearsed regularly to support children in counting efficiently with groups of objects or the rows of an array.</li><li>• Note that as per the calculation policy, the array for <math>a \div b = c</math> should be represented as <math>a</math> objects grouped into <math>a</math> rows by <math>b</math> columns (i.e. <math>c</math> across and <math>b</math> down). You should introduce this now, even though the formal array is challenging for stage 1, so that there is no 'unlearning' to be done later.</li><li>• When approaching problems, try to look at the language that tells you what you should be doing - model the calculation as you read the</li></ul>

This can lead children to half an odd number and give a whole number answer.

problem rather than all at the end. Try to represent problems in different ways (grouped objects, bead strings, arrays) to develop flexibility in the children.

#### Key Assessment Checklist

1. I can represent a division problem by sharing.
2. I can represent a division problem by grouping.
3. I can represent a division problem by grouping objects into an array.
4. I can identify and then solve a division problem using sharing or grouping with support.
5. I can solve one-step problems involving either multiplication or division, recognising which operation is required.

Stage 1	Unit 14: Describing Position			
<b>7 learning hours</b>	In this unit pupils explore how we can communicate position and movement mathematically. They look at transformations from simple turns to reflection/rotation/enlargement/translations up to similar shapes generated by enlargements, co-ordinate systems and ultimately vectors			
Prior Learning	Core Learning	Learning Leads to...		
30-50 months > Uses positional language 40-60 months > Can describe relative position of objects such as <i>behind</i> or <i>next to</i> ELG: > Children use everyday language to talk about position to compare objects and solve problems. They recognise and describe patterns.	> <b>describe position, direction and movement, including whole, half, quarter and three-quarter turns</b>	> use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise)		
Exemplification		Vocabulary		
1. a) Look at this picture:  Complete these sentences: The circle is above the ..... The circle is next to the ..... The square is ..... the triangle  b) Which of these turns shows a quarter turn? 		<table border="1"> <tr> <td data-bbox="1615 756 1861 1098">           where position  above below next to beside under inside outside  in front of on top of to the left of to the right of behind between         </td> <td data-bbox="1861 756 2098 1098">           left right forward backwards  turn half quarter three quarter (clockwise) (anticlockwise)         </td> </tr> </table>	where position  above below next to beside under inside outside  in front of on top of to the left of to the right of behind between	left right forward backwards  turn half quarter three quarter (clockwise) (anticlockwise)
where position  above below next to beside under inside outside  in front of on top of to the left of to the right of behind between	left right forward backwards  turn half quarter three quarter (clockwise) (anticlockwise)			

Representation	Fluency	Probing Questions
<p><b>Position: 3D</b></p> <ul style="list-style-type: none"> <li>Playing hide and seek games where children hide and the others must describe where they are hiding. For example: 'inside the cupboard' or 'behind the desk' or 'next to the coats'.</li> <li>Playing bear and box to place the bear in the correct place (can be done as a whole class if each child has a toy and a box]. For example: put the bear on top of the box or put the bear in front of the box</li> </ul>	<p>1. Describe the position of a 3D object relative to another</p> <ul style="list-style-type: none"> <li>above, below, under, on top of</li> <li>in front of, behind</li> <li>next to, beside</li> <li>between</li> <li>inside, outside</li> </ul>	<p>Show me an object that is above (another object)</p> <p>Show me the second house on the right</p> <p>What's the same and what's different? above; left; right; below; next to; under</p>
<p><b>Position: 2D</b></p> <ul style="list-style-type: none"> <li>Playing 'find the shape' games by finding the shape/object from a given picture that is in a given position e.g. above the square. Children can also be the describer and give instructions to others to guess their shape. (The image in Further Extension is useful for this).</li> </ul>	<p>2. Describe the position of a 2D object relative to another</p> <ul style="list-style-type: none"> <li>above/below</li> <li>next to, beside</li> <li>to the left of, to the right of</li> </ul>	<p>Show me a word that can complete this sentence (the triangle is below the ....)</p> <p>Show me the third shape on the left hand side</p> <p>Convince me the square is above the triangle</p>
<p><b>Position: 3D – left and right</b></p> <ul style="list-style-type: none"> <li>Playing bear and box to place the bear in the correct place (can be done as a whole class if each child has a toy and a box] using lefts and rights For example: put the bear on top of the box or put the bear in front of the box</li> <li>Sitting at a dining table and describing where people are in relation to them (e.g. on my left...)</li> </ul>	<p>3. Describe the position of a 3D object using left and right</p> <ul style="list-style-type: none"> <li>describe position of one object compared to another from own viewpoint e.g. the bear is to the left of the box</li> <li>name things that are on their left</li> <li>name things that are on their right</li> <li>understand that if they change position, the things that are on their left/right will be different</li> </ul>	
<p><b>Direction and Turn</b></p> <ul style="list-style-type: none"> <li>Turning (people/themselves) through quarter turns and chanting <math>\frac{1}{4}</math> turn, <math>\frac{1}{2}</math> turn, <math>\frac{3}{4}</math> turn and so on. Changing direction from clockwise to anticlockwise. This can also be done with clock hands.</li> </ul>	<p>4. Describe and carry out turns</p> <ul style="list-style-type: none"> <li>recognise a whole turn</li> <li>carry out a whole turn</li> <li>recognise a half turn</li> <li>carry out a half turn (in either direction)</li> <li>recognise a quarter turn</li> </ul>	<p>Show me a quarter turn Show me a half turn Show me a three quarter turn</p> <p>Convince me that you have turned three quarters of a turn</p>

	<ul style="list-style-type: none"> <li>• carry out a quarter turn (in either direction)</li> <li>• recognise a three quarter turn</li> <li>• carry out a three quarter turn</li> <li>• ext: begin to use the words clockwise and anticlockwise to describe and carry out turns</li> <li>• understand a diagram showing a turn with an arrow</li> </ul>	<p>What's the same and what's different? half turn; two quarter turns; face the opposite way</p>
<p><b>Directions</b></p> <ul style="list-style-type: none"> <li>• Following instructions to find the treasure e.g. walk forwards 10 steps. Then turn a quarter turn to the right.</li> <li>• Giving their own instructions to find an item to a partner or teacher</li> <li>• Using Beebots or Scratch to experiment with making other objects follow paths and carry out turns</li> </ul>	<p>5. Describe or carry out a single movement</p> <ul style="list-style-type: none"> <li>• in a straight line             <ul style="list-style-type: none"> <li>○ forwards</li> <li>○ backwards</li> </ul> </li> <li>• turn left / turn right</li> <li>• <math>\frac{1}{4}</math> or <math>\frac{1}{2}</math> or <math>\frac{3}{4}</math> turn</li> </ul>	<p>What's the same and what's different? forwards; backwards; left; right</p> <p>Always, Sometimes, Never? Turns can only take place in a clockwise direction.</p> <p>Always, Sometimes, Never? You can't turn less than a quarter turn.</p>
	<p>6. Describe a pathway that an object moves down (2 or more steps)</p> <ul style="list-style-type: none"> <li>• describe a journey (forwards, backwards, turn left, turn right)</li> <li>• describe a journey including <math>\frac{1}{4}</math> and <math>\frac{1}{2}</math> turns (and <math>\frac{3}{4}</math>)</li> <li>• give instructions for a path or journey in real life</li> <li>• ext: give instructions for a path or journey from a diagram</li> </ul>	<p>Show me how you can get from the start to the finish on this map</p>
	<p>7. Understand the links between turns</p> <ul style="list-style-type: none"> <li>• realise that two quarter turns in the same direction make a half turn</li> <li>• realise that three quarter turns in the same direction make a <math>\frac{3}{4}</math> turn</li> <li>• recognise that the direction does not matter for a half turn</li> <li>• recognise that a <math>\frac{1}{4}</math> turn in one direction is the same as a <math>\frac{3}{4}</math> turn in the other</li> </ul>	<p>Show me that a half turn can be the same as two quarter turns</p> <p>Convince me that you can make a half a turn either way and get to the same place</p> <p>What's the same and what's different? three-quarter turn; a half turn and a quarter turn; a quarter turn the other way</p> <p>Always, Sometimes, Never? It doesn't matter which way you turn, you will get to the same place.</p>

### Further Extension

### Rich and Sophisticated Tasks

1.  
a)  
Identify the position of each item.  
Top, middle or bottom?  
First, second or third?  
Left or right?



The cups are in the middle row and third from the left.

The shapes are in the  row and  from the left.

The rulers are in the  row and  from the right.

The maths books are in the  row and  from the right.

Describe the position of other items.

b)

- Which drawer will Ziggy open?  
You may ask him four questions to identify the drawer.  
He can only answer 'Yes' or 'No'.  
Which four questions would you ask?  
Explain your reasoning.

Describe position, direction and movement, including whole, half, quarter and three-quarter turns

- NRICH: [Tangram Tangle](#) \*\*\* G  
NRICH: [Olympic Rings](#) \*\* I  
NRICH: [2 Rings](#) \* I  
NRICH: [Turning](#) \* I

### Misconceptions

### Teacher Guidance and Notes

Children will frequently confuse left and right directions. This is particularly challenging when the start point is not aligned with them as readers/viewers.

Some children may believe that all turns have to be clockwise and struggle to comprehend an anticlockwise turn, sometimes to the extent where they find it hard to turn in that direction themselves physically.

- This unit is children's first formal introduction to the mathematical language of position and direction; however, they will have been using this language in other contexts frequently.
- Ensure children are able to describe individual movements clearly and correctly e.g. turn left or make a half turn or move forward three steps. before looking at a path that combines a sequence of moves


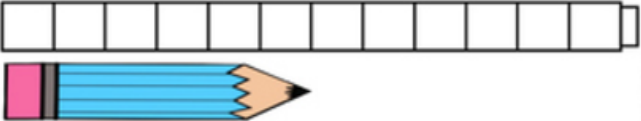
When turning, some children may lose count of where they started and/or how much (how many quarters) they have turned. They may need to mark the starting point and count as they turn.

- Teachers will need to ensure that the pupils can distinguish between quarter turns, half turns and three-quarter turns and that the movement takes place in both directions (at this stage this is more important than naming the directions, which is Stage 2 content). Help students to mark their starting points clearly so they can go back and test their descriptions.
- Relate the language of half and, where possible, quarter to work completed earlier on fractions and on time. Show representations of a half e.g. half a circle and link this to the turn that is taking place.

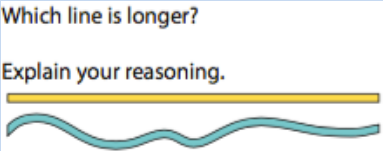


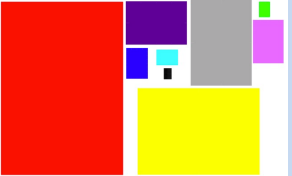

#### Key Assessment Checklist

1. I can describe the position of an object in relation to another using the language above, below and next to.
2. I can describe the position of an object in relation to another using 'left' and 'right'.
3. I can describe the pathway when an object has moved.
4. I can describe the amount of turn an object takes as a half turn, a quarter turn or a three quarter turn.
5. I can recognise that quarter turns perform the same action as turning right or left.
6. I can recognise that multiples of quarter turns are equivalent to half and three-quarter turns




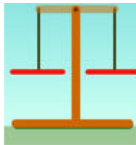
Stage 1	Unit 15: Measuring and Estimating	
<p><b>8 learning hours</b></p>	<p>In this unit children and students explore measurement, standard units of measure and conversions between these units. At secondary level this extends to investigating compound measures such as density and pressure.</p>	
Prior Learning	Core Learning	Learning Leads to...
<ul style="list-style-type: none"> <li>&gt; DM: 40-60+ months:               <ul style="list-style-type: none"> <li>o Orders two or three items by length or height.</li> <li>o Orders two items by weight or capacity.</li> <li>o Uses everyday language related to time.</li> <li>o Orders and sequences familiar events.</li> <li>o Measures short periods of time in simple ways.</li> </ul> </li> <li>&gt; ELG: use everyday language to talk about size, weight, capacity, and time</li> </ul>	<ul style="list-style-type: none"> <li>&gt; <b>compare, describe and solve practical problems for:</b> <ul style="list-style-type: none"> <li>o <b>lengths and heights</b> [for example, long/short, longer/shorter, tall/short, double/half]</li> <li>o <b>mass/weight</b> [for example, heavy/light, heavier than, lighter than]</li> <li>o <b>capacity and volume</b> [for example, full/empty, more than, less than, half, half full, quarter]</li> <li>o <b>time</b> [for example, quicker, slower, earlier, later]</li> </ul> </li> <li>&gt; <b>measure and begin to record the following:</b> <ul style="list-style-type: none"> <li>o <b>lengths and heights</b></li> <li>o <b>mass/weight</b></li> <li>o <b>capacity and volume</b></li> <li>o <b>time (hours, minutes, seconds)</b></li> </ul> </li> </ul>	<ul style="list-style-type: none"> <li>&gt; choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels</li> <li>&gt; compare and order lengths, mass, volume/capacity and record the results using &gt;, &lt; and =</li> </ul>
Key Assessment Checklist		
<ol style="list-style-type: none"> <li>1. I can describe and compare measures of distance (length or height) to solve a problem.</li> <li>2. I can describe and compare measures of mass and weight to solve a problem.</li> <li>3. I can describe and compare measures of capacity (or volume) to solve a problem.</li> <li>4. I can describe and compare measures of time to solve a problem.</li> <li>5. I can measure distances, masses and capacities using non-standard units.</li> <li>6. I can make comparisons between objects using my measurements.</li> <li>7. I can measure distances, masses, capacities and times using standard units.</li> <li>8. I can make comparisons between objects using my measurements in standard units.</li> </ol>		


Exemplification	Vocabulary	
<p>1. Circle the best answer :</p> <p>a) The length of a car is about    2 centimetres    2 metres    2 miles    2 kilograms</p> <p>b) The capacity of a water bottle is about    5 millilitres    500 millilitres    5 litres    5 metres</p> <p>c) The length of a maths lesson is about    60 seconds    60 minutes    60 hours    60 days    60 grams</p> <p>2.</p> <p>a) Use a ruler to measure the length of this line in centimetres</p>  <p>b) Mel is measuring a pencil using blocks. How long is the pencil in blocks?</p> 	<p>length</p> <p>height</p> <p>width</p> <p>long/short</p> <p>longer/shorter than...</p> <p>tall/short</p> <p>taller/shorter than ...</p> <p>double/half</p> <p>weight</p> <p>mass</p> <p>heavy/light</p> <p>heavier than/lighter than</p> <p>capacity</p> <p>volume</p> <p>contains</p> <p>full/empty</p> <p>more than/less than</p> <p>half/ half full/ quarter</p>	<p>full</p> <p>faster/quicker/</p> <p>slower than</p> <p>units</p> <p>metre</p> <p>kilogram</p> <p>litre</p> <p>hour</p> <p>minute</p> <p>second</p> <p>day</p> <p>measure</p> <p>metre stick</p> <p>ruler</p> <p>scales</p> <p>jug</p> <p>container</p> <p>stopwatch</p> <p>clock</p> <p>timer</p>

Flow 1: Length ~ 3 hours		
Representation	Fluency	Probing Questions
<ul style="list-style-type: none"> <li>Measuring lengths using objects e.g. cubes, counting bears, straws, ...</li> <li>Measuring heights using blocks, hands, books, ....</li> <li>Exploring the use of rulers marked in centimetres – measuring and seeing if you agree with your partner</li> <li>Exploring other devices including measuring tapes, tape measures, metre sticks and trundle wheels for measuring lengths and heights</li> <li>Estimating a length/height and then checking by measuring</li> <li>Deciding which is the best piece of equipment for measuring different lengths (give children different equipment to measure the same thing and see which is the easiest/hardest)</li> </ul>	<ol style="list-style-type: none"> <li>Use the language and ideas of length               <ul style="list-style-type: none"> <li>Know that the length measures how far it is from one of an object to the other (in a straight line)</li> <li>Use the language of long, tall and short to describe objects</li> <li>Given two objects/drawn objects, say which is longer and which is shorter</li> <li>Given two objects, say which is taller and which is shorter</li> <li>Order 3 or more objects from tallest to shortest or from longest to shortest (or vice versa)</li> </ul> </li> <li>Measure lengths with non-standard units               <ul style="list-style-type: none"> <li>Measure lengths and heights of <i>objects</i> using given apparatus e.g. cubes and say how many cubes long it is; record these measurements</li> <li>Measure lengths of <i>drawn items</i> using given apparatus e.g. cubes and say how many cubes long it is; record these measurements</li> <li>Measures lengths of <i>drawn items</i> where <i>apparatus has been drawn alongside</i> (e.g. in exemp q 2)</li> </ul> </li> <li>Measure lengths using simple standard units               <ul style="list-style-type: none"> <li>measure short length in centimetres using a ruler</li> <li>measure a longer length in metres using a metre rule, tape measure or trundle wheel</li> <li>record the measurement with the units</li> <li>ext: measure a longer length in centimetres using a tape measure, measuring tape or metre rule</li> </ul> </li> <li>Compare lengths and order them               <ul style="list-style-type: none"> <li>Given two lengths (measurements), say which is longer and which is shorter</li> <li>Given two heights (measurements), say which is taller and which is shorter</li> <li>Given three lengths or heights, order them as longest/tallest, shorter and shortest (or other way round i.e. shortest, longer/taller and longest/tallest)</li> </ul> </li> </ol>	<p>Show me</p> <ul style="list-style-type: none"> <li>... something that is longer than a pencil</li> <li>... something that is shorter than a pencil</li> <li>... something that is longer than a metre</li> </ul> <p>Convince me that this mat is twice as long as the other mat</p> <p>What's the same and what's different? Length Height (Width)</p> <p>True or False? You can use a metre stick to measure any object</p>

	<ul style="list-style-type: none"><li>• Order lengths in a mix of centimetres and metres</li><li>• Order four or more lengths and heights (measurements)</li><li>• Combine above skills to measure, record and order lengths or heights, using correct words to describe</li></ul>	
<p><b>Further Extension</b></p> <p>1. Which line is longer? Explain your reasoning.</p>  <p>2. A long brick is twice the length of a short brick. Which is longer: 2 long bricks or 3 short bricks? 3 long bricks or 5 short bricks?</p> 	<p><b>Rich Tasks</b></p> <p>How could you find out the height of a tree?</p>  <p><b>NRICH: <a href="#">How Tall?</a> * I</b></p> <p>How can you order these rectangles from smallest to largest?</p>  <p>What do you notice about the lengths and heights of the rectangles? <b>NRICH: <a href="#">Different Sizes</a> * P I</b></p> <p><b>Can You Do it Too?</b></p> <p>Age 5 to 7 **</p> <p>Here are some pictures of people throwing things in Olympic events. The first one shows a man throwing a hammer and the second shows a man throwing a discus.</p>  <p>They can throw them a long way. A recent Olympic record for throwing the hammer was about 80 adult paces and for throwing the discus was about 70 adult paces.</p> <p>How far can you throw a beanbag?</p> <p><b>NRICH: <a href="#">Can You Do it Too?</a> ** G</b></p>	<p><b>Misconceptions</b></p> <p>When comparing, children sometimes struggle to find the difference between their measurements, although they may be able to say which is larger and which is smaller.</p> <p>Children may use longer, shorter etc... as a singular adjective rather than to compare two amounts. They may muddle up taller/longer/wider etc. because there are a number of different words for different distances.</p>

Flow 2: Mass ~ 2 learning hours		
Representation	Fluency	Probing Questions
<ul style="list-style-type: none"> <li>Exploring the use of balance scales, electronic scales, scales with a dial scale, hanging balances for measuring masses</li> <li>Estimating a mass and then checking by measuring</li> <li>Deciding which is the best piece of equipment for measuring different masses</li> </ul>	<p>5. Use the language and ideas of mass</p> <ul style="list-style-type: none"> <li>Know that the mass measures how heavy it is</li> <li>Use the language of heavy and light to describe objects</li> <li>Given two objects, say which is heavier and which is lighter</li> <li>Order 3 or more objects from heaviest to lightest (or vice versa)</li> </ul> <p>6. Measure masses with</p> <ul style="list-style-type: none"> <li>Use electronic scales to measure the mass of an object in grams or kilograms</li> <li>Use dial scales to measure the mass of an object in kilograms (only whole numbers)</li> <li>Use hanging balances to measure the mass of an object</li> <li>Use balance scales to measure the mass of an object in non-standard units e.g. apples</li> <li>Use balance scales and weights to measure the mass of an object in grams or kilograms</li> </ul> <p>7. Compare masses and order them</p> <ul style="list-style-type: none"> <li>Given two masses (measurements), say which is heavier and which is shorter</li> <li>Given three masses, order them as heaviest, lighter, lightest oe</li> <li>Order four or more masses</li> <li>Order masses where there is a mix of units</li> <li>Combine above skills to measure, record and order masses using correct words to describe this</li> </ul>	<p>Show me ... something that is lighter than you ... something that is heavier than a kilogram</p> <p>Show me how you could see which is heavier: a toy car or a toy dinosaur.</p> <p>Show me something you could use to measure the mass of this ball</p> <p>Convince me that that this book is heavier than a kilogram</p> <p>What's the same and what's different? weight and size</p> <p>What's the same and what's different? mass and weight</p>

Further Extension	Rich Tasks	Misconceptions
<p>1. Look at these balance scales. There are five cars on one side. The doll weighs the same as how many cars?</p>  <p>2. Which toy is heavier?</p>  <p>If you added a toy car to the teddy, what would happen to the scales? Explain your reasoning.</p> <p>3. Which of these statements is true?</p> <ul style="list-style-type: none"><li>■ The dinosaur is lighter than the robot.</li><li>■ The robot is lighter than the dinosaur.</li><li>■ The dinosaur and robot weigh the same.</li></ul>  <p>Explain your reasoning.</p>	<p>Put these masses in order – you might need to be creative to find some of them out!</p>  <p><b>Mass</b> Of a blown-up balloon Of a bar of chocolate Of a loaf of bread Of your teacher</p> <p>Investigate a set of scales – can you measure mass in something that isn't kilograms or grams? What is it? Measure some objects with this new unit. What can you find out?</p>	<p>Children may use lighter, heavier, ... as a singular adjective rather than to compare two amounts.</p> <p>Children find it hard to estimate and imagine weight - they may not be able to guess how heavy something is. Their concept of weight may be limited and so they may not really understand what a mass (weight) is.</p>

Flow 3: Capacity ~ 2 learning hours		
Representation	Fluency	Probing Questions
<ul style="list-style-type: none"> <li>Exploring how many cups/jugs of water/sand/cubes fit inside an object – consider which of these is the best to use (why are cubes tricky?!)</li> <li>Bottles task: <b>NRICH: Bottles (1) * P</b></li> </ul>  <ul style="list-style-type: none"> <li>Using a litre jug to measure capacity in litres by filling an object repeatedly with 1 litre of water/sand</li> <li>Exploring the use of measuring cylinders, measuring jugs, litre containers, larger containers for measuring capacities</li> <li>Estimating a capacity and then checking by measuring</li> <li>Deciding which is the best piece of equipment for measuring different capacities</li> </ul>	<p>8. Use the language and ideas of capacity</p> <ul style="list-style-type: none"> <li>Know that the capacity measures how much space there is inside an object</li> <li>Understand that measuring capacity is easiest with a liquid or solid that can change shape e.g. sand because it does not leave gaps</li> <li>Use the language of full, empty, half full/half empty describe objects containing something</li> <li>Given two objects, say which is bigger/holds more and which is smaller/holds less</li> <li>Describe a capacity as double another or half another e.g. holds twice as much</li> <li>Order 3 or more objects by their capacity</li> </ul> <p>9. Measure capacity</p> <ul style="list-style-type: none"> <li>Use cups or jugs or other given apparatus to find the capacity of an object</li> <li>Find the capacity of two or three objects using the same (non-standard) apparatus and use this to order them by capacity</li> <li>Use a 1 litre measuring jug to find the capacity of an object in litres</li> <li>Use a measuring cylinder or jug to find the capacity of an object by reading from a simple scale</li> <li>Ext: start to use millilitres for familiar objects e.g. multiples of 10 or 100, depending on number sense</li> </ul> <p>10. Compare capacities and order them</p> <ul style="list-style-type: none"> <li>Given two capacities (measurements), say which is heavier and which is shorter</li> <li>Given three capacities, order them</li> <li>Order four or more capacities</li> <li>Order capacities where there is a mix of units</li> <li>Combine above skills to measure, record and order capacities using correct words to describe this</li> </ul>	<p>Show me</p> <ul style="list-style-type: none"> <li>... a picture of something that is half full</li> <li>... a picture of something that is a quarter full</li> <li>... something you could use to measure the capacity of this bowl</li> </ul> <p>Convince me that half on one container can be more than a different container which is full (pictures needed)</p> <p>Convince me that a short wide glass can contain more water than a tall thin glass</p> <p>What's the same and what's different? Measuring capacity with water Measuring capacity with sand Measuring capacity with cubes</p> <p>What's the same and what's different? capacity; space; volume; amount</p> <p>Always, Sometimes, Never? Bigger objects contain more than smaller ones</p> <p>Always, Sometimes, Never? If the height of the liquid is the same in each container, then they contain the same amount.</p>

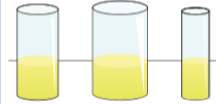
## Further Extension

1.

### CAPACITY

Captain Conjecture says, 'All of the glasses contain the same quantity of lemonade.'

Do you agree?



Explain your reasoning.



2.

Sid has a full bottle of drink. He pours it into a jug.

Which has the greater capacity, the bottle or the jug?



3.

Dave has a 1 litre and a 2 litre bottle. He pours the water from the small bottle into the large bottle.

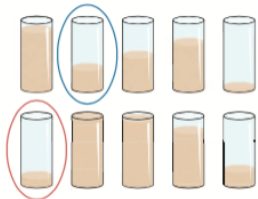
Mark where the water comes to on the large bottle.



4.

Point to a glass which is about half as full as the glass in the red oval?

Can you point to a glass which is about twice as full as the glass in the blue oval?



## Rich Tasks

### Thirsty?

Age 5 to 7 ★

This problem has been designed to work on in a group of about four. For more details about how you might go about doing this, please read the [Teachers' Notes](#).



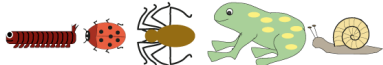
You will need to print off [these eight cards](#), which have pictures of glasses of orange and blackcurrant juice on them. You need one set of these cards for your group.

NRICH: [Thirsty?!](#)

## Misconceptions

Children find it hard to estimate and imagine capacity - they may not be able to guess how many jugs of water will fill the bowl etc.

There are often misconceptions regarding conservation of volume, ie short wide glass contains less than a tall thin glass.

Flow 4: Time ~ 1-2 learning hours		
Representation	Fluency	Probing Questions
<ul style="list-style-type: none"> <li>Exploring measuring time using counting and realising that this brings in error and variation because everyone counts at different speed</li> <li>Exploring equipment such as timers, stopwatches, clocks etc for measuring time</li> </ul>	<p>11. Use the language and ideas of time (links to Unit 12)</p> <ul style="list-style-type: none"> <li>Know that time measures how long something takes</li> <li>Recognise that we can measure time in seconds, minutes, hours, days, weeks, years etc</li> <li>Use the language of faster/slower and longer/shorter to describe times</li> <li>Estimate a time roughly</li> <li>Suggest an appropriate unit for a time</li> </ul> <p>12. Measure short times with</p> <ul style="list-style-type: none"> <li>A clock by counting the seconds alongside</li> <li>An electronic stopwatch using the stop/go button</li> <li>A timer (analogue) by reading the number of seconds or minutes from the scale e.g. an egg timer for minutes</li> </ul> <p>13. Compare times and order them</p> <ul style="list-style-type: none"> <li>Given two times say which is longer/slower and which is quicker/faster</li> <li>Given three times, order them as fastest, slower, slowest oe</li> <li>Order four or more times</li> <li>Order times where there is a mix of units (but no bridging needed)</li> <li>Combine above skills to measure, record and order times using correct words to describe this</li> </ul>	<p>Show me something you could use to measure the time it takes to build this tower</p> <p>What's the same and what's different? metre stick; scales; timer</p> <p>What's the same and what's different? time and age</p> <p>Always, Sometimes, Never? Longer times are better</p>
Further Extension	Rich Tasks	Misconceptions
<p>1. I walk to school every day. On Monday my journey takes 10 minutes. On Tuesday I walk more slowly. Does my journey take more or less time than on Monday?</p> <p>Explain your answer.</p> <p>On Wednesday it takes me 8 minutes to walk to school. On which of the 3 days do I walk quickest? On which of the 3 days do I walk slowest?</p> <p>Explain your reasoning.</p>	<p><i>How long does it take you to walk 10m? So how long would it take you to walk 20m? 50m? 100m? 1 km? How long would it take you to walk 5 km?</i></p> <p><b>The Animals' Sports Day</b></p> <p><small>Age 5 to 7 ★</small></p> <p><small>One day five small animals in my garden decided to have a sports day.</small></p>  <p><b>NRICH: <a href="#">The Animals' Sports Day</a> * I</b></p>	<p>Children get confused between different types of measurement e.g. they don't know whether they need to measure a distance or a mass or a capacity to answer the question</p> <p>Children may have issues with bridging units of time in base 10 when recording time.</p>

### Teacher Guidance

- This unit is based around both the comparative language of measurement and the concept of equivalence (to non-standard and some standard units). Time should be spent encouraging children to explore which is longer, taller, heavier, of greater capacity, fuller etc. in order to expose them to a range of objects and language, Note that capacity is particularly neglected in general and therefore should be emphasised so that children's sense of volume is developed. The opportunity to explore a range of containers, objects and measuring equipment should be provided.
- Be sure that children know what a mass (weight) and a capacity actually are - get them to hold items to 'feel' different 'heavinesses' and to fill containers and transfer the contents to 'feel' capacity.
- Make time to explore non-standard units e.g. lengths of plasticine, cubits, blocks, jugs of water - children should be encouraged to measure with these as well as with standard units e.g. metre sticks. This will help them to understand that there are a number of different ways to measure any quantity and the standard units are just the well-known ways. For example, you could devise a class unit of length e.g. a block and use this to estimate the length of the classroom, the height of the door etc. You can also think of items which are hard to measure with the block to develop the idea that we need a range of units of measurement to help us measure things that are both big and small. Try using different units to show that the capacity of one container is double that of a smaller container - it doesn't matter which units are used, this will still be discoverable!
- When looking at standard units, focus initially on metres, centimetres, kilograms, litres and minutes/seconds. Be prepared to look at kilometres, grams, tonnes, millilitres and hours.